

magnetic current $j^\mu = \varepsilon^{\mu\nu} \partial_\nu \varphi$

finite energy $j^\mu \underset{x \rightarrow \infty}{\rightsquigarrow} 0$

conserved $\partial_\mu j^\mu = 0$

$$\partial_\mu j^\mu = \partial_\mu \varepsilon^{\mu\nu} \partial_\nu \varphi = \underbrace{\varepsilon^{\mu\nu}}_{\text{asym}} \underbrace{\partial_\mu \partial_\nu \varphi}_{\text{sym}} = 0$$

$$\text{magnetic charge } \mathcal{J} = \int_{\mathbb{R}^d} dx^{t,x} j^0 \rightsquigarrow \int_{r\mathbb{B}_d} dx^{t,x} j^0$$

conserved $\partial_0 \mathcal{J} = 0$

$$\partial_0 \mathcal{J} \rightsquigarrow \partial_0 \int_{r\mathbb{B}_d} j^0 = \int_{r\mathbb{B}_d} \partial_0 j^0 = - \int_{r\mathbb{B}_d} \partial_m j^m = - \int_{r\mathbb{S}_{d-1}} dS_m j^m \underset{j^m \rightarrow 0}{\rightsquigarrow} 0$$