$$\begin{bmatrix}\varphi \\ =\end{bmatrix} \frac{\overbrace{\partial_t\varphi}^{2} - \overbrace{\partial_x\varphi}^{2}}{2} + \cos\varphi - 1$$

$$\mathbb{R}^{1:1} \xrightarrow{\varphi} \mathbb{R} \text{ real scalar field}$$

$$\text{motion } \partial_t^2\varphi - \partial_x^2\varphi = \partial_\mu\partial^\mu \mathcal{L}_\varphi = \partial\mathcal{L}_\varphi = -\sin\varphi$$

$$\text{constant solutions } n\pi{:}n \in \mathbb{Z}$$

$$\text{energy-density } {}_0\mathcal{L}_\varphi^0 = \underbrace{\partial_0\varphi}\,\underbrace{\partial^0\mathcal{L}_\varphi} - \mathcal{L}_\varphi = \overbrace{\partial_0\varphi}^{2} - \mathcal{L}_\varphi = \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + 1 - \cos\varphi$$

$$V(\varphi) = 1 - \cos\varphi$$

$$\underline{V}(\varphi) = \sin\varphi$$

$$\underline{\underline{V}}(\varphi) = \cos\varphi$$

$$\underline{\underline{V}}(\pi n) = \cos n\pi = (-1)^n$$

$$\begin{cases} \underline{\underline{V}}\left(\underbrace{2n+1}\pi\right) = -1 < 0 & \Longrightarrow \text{max=anti-vacuum} \\ \underline{\underline{V}}(2n\pi) = 1 > 0 & \Longrightarrow \text{min=vacuum} \end{cases}$$

$$\text{vacuum density } {}_0\mathcal{L}_v^0 = f/4\,m^4/f^2 - m^2/2\,m^2/f = -m^4/4f$$

$${}_0\mathcal{L}_\varphi^0 - {}_0\mathcal{L}_v^0 = \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + f\varphi^4/4 - m^2\varphi^2/2 + m^4/4f = \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + f\left(\varphi^2 - \overset{2}{m^2/f}\right)/4$$

$${}_0L_\varphi^0 - {}_0L_v^0 = \int_{\mathbb{R}}^{dx} \frac{\overbrace{\partial_t\varphi}^{2} + \overbrace{\partial_x\varphi}^{2}}{2} + f\left(\varphi^2 - \overset{2}{m^2/f}\right)/4$$

static solutions

$$\partial_t\varphi = 0 \Longrightarrow \partial_x^2\varphi - \sin\varphi = 0 \Longrightarrow \partial_x\left(\overbrace{\partial_x\varphi}^{2} + 2\cos\varphi\right) = 2\,\partial_x\varphi\underbrace{\partial_x^2\varphi - \sin\varphi} = 0$$

$$\Longrightarrow \overbrace{\partial_x\varphi}^{2} + 2\cos\varphi = 2C \Longrightarrow \partial_x\varphi = \sqrt{2(C - \cos\varphi)} = \frac{d\varphi}{dx}$$

$$\Longrightarrow dx = \frac{d\varphi}{\sqrt{2(C - \cos\varphi)}}$$

static finite energy solutions

$$\int_{\mathbb{R}}^{dx} \frac{\overbrace{\partial_x\varphi}^{2}}{2} + 1 - \cos\varphi < \infty \Longrightarrow$$

$$\int_{\mathbb{R}}^{dx} \overbrace{\partial_x\varphi}^{2} < \infty \Longrightarrow \partial_x \varphi \xrightarrow[x \to \infty]{} 0$$

$$\int_{\mathbb{R}}^{dx} 1 - \cos\varphi < \infty \Longrightarrow \cos {}^{x}\varphi \xrightarrow[x \to \infty]{} 1 \Longrightarrow {}^{x}\varphi \xrightarrow[x \to \infty]{} 2\pi n$$

$$C = \cos\varphi + \frac{\overbrace{\partial_x\varphi}^{2}}{2} \xrightarrow[x \to \infty]{} 1 \Longrightarrow C = 1 \Longrightarrow \overbrace{\partial_x\varphi}^{2} = f\,\varphi^4/2 - m^2\,\varphi^2 + m^4/2f = f\left(\varphi^2 - \overset{2}{m^2}/f\right)/2$$

$$\Longrightarrow \partial_x\varphi = \sqrt{f/2}\left(\varphi^2 - m^2/f\right) \Longrightarrow \sqrt{f/2}dx = \frac{d\varphi}{\varphi^2 - m^2/f} \Longrightarrow \sqrt{f/2}\left(x - x_0\right) = \int \frac{d\varphi}{\varphi^2 - m^2/f}$$

$$\Longrightarrow {}^{x}\varphi = \geqslant -4\overset{-1}{\tan}\exp\left(x - x_0\right)$$