$$e_{\mathbb{C}}\,(x) = \; e^{2\pi i x/\ell}$$

$$\underset{\tau}{0|\ell}\,\underset{\infty}{\triangle}\,\mathfrak{t}^{\mathbb{C}}\,\bowtie\mathbb{R}\;\xrightarrow[()^{\mu}]{{}_{0}()^{\mu}}\;\mathbb{R}\!: \quad 0|\ell\,\underset{\infty}{\triangle}\,\mathfrak{t}^{\mathbb{C}}\,\bowtie\mathbb{R}\;\xrightarrow[()^{\mu}_{-n}]{()^{\mu}_{n}}\;\mathbb{C}\!: \quad \underset{\tau}{0|\ell}\,\underset{\infty}{\triangle}\,\mathfrak{t}^{\mathbb{C}}\,\bowtie\mathbb{R}\;\xrightarrow[\overset{\mathbb{R}}{()}{}^{\mu}_{-n}:\overset{\mathbb{I}}{()}{}^{\mu}_{-n}]{\overset{\mathbb{R}}{()}{}^{\mu}_{n}:\overset{\mathbb{I}}{()}{}^{\mu}_{n}}\;\mathbb{R}$$

$${}^{\sigma}(\tau)^{\mu} - ()^{\mu} - \frac{\alpha'}{\not\ell}\,{}_{0}()^{\mu}\,\tau = i\sqrt{\alpha'/2}\sum_{n\neq 0}^{\overline{n}^{-\,1/2}} e_{\mathbb{C}}^{\sigma n}\,\underbrace{()^{\mu}_{n}e_{\mathbb{C}}^{-\tau\overline{n}} - \overset{*}{()}{}^{\mu}_{-n}e_{\mathbb{C}}^{\tau\overline{n}}} = i\sqrt{\alpha'/2}\sum_{n\neq 0}^{\overline{n}^{-\,1/2}}\,\underbrace{()^{\mu}_{n}e_{\mathbb{C}}^{\sigma n\,-\,\tau\overline{n}} - \overset{*}{()}{}^{\mu}_{n}e_{\mathbb{C}}^{\tau\overline{n}\,-\,\sigma n}}$$

$${}^{\tau\sigma}X^{\mu} = \; X^{\mu} + \frac{\alpha'}{\not\ell}\,{}_{0}X^{\mu}\,\tau + i\sqrt{\alpha'/2}\sum_{n\neq 0}^{1/n}\alpha^{\mu}_{n}\,e_{\mathbb{C}}^{(\sigma\,-\,\tau)\,n} + \underset{\sim}{\alpha}{}^{\mu}_{n}\,e_{\mathbb{C}}^{-\,(\sigma\,+\,\tau)\,n}$$

$$\sum_{n\neq 0}^{1/n}\alpha^{\mu}_{n}\,e_{\mathbb{C}}^{(\sigma\,-\,\tau)\,n} + \underset{\sim}{\alpha}{}^{\mu}_{n}\,e_{\mathbb{C}}^{-\,(\sigma\,+\,\tau)\,n} = \sum_{n\neq 0}^{1/n}\alpha^{\mu}_{n}\,e_{\mathbb{C}}^{\sigma n}\,e_{\mathbb{C}}^{-\tau n} + \underset{\sim}{\alpha}{}^{\mu}_{n}\,e_{\mathbb{C}}^{-\sigma n}\,e_{\mathbb{C}}^{-\tau n}$$

$$= \sum_{n>0}^{1/n}\alpha^{\mu}_{n}\,e_{\mathbb{C}}^{\sigma n}\,e_{\mathbb{C}}^{-\tau n} + \underset{\sim}{\alpha}{}^{\mu}_{n}\,e_{\mathbb{C}}^{-\sigma n}\,e_{\mathbb{C}}^{-\tau n} - \alpha^{\mu}_{-n}\,e_{\mathbb{C}}^{-\sigma n}\,e_{\mathbb{C}}^{\tau n} - \underset{\sim}{\alpha}{}^{\mu}_{-n}\,e_{\mathbb{C}}^{\sigma n}\,e_{\mathbb{C}}^{\tau n}$$

$$= \sum_{n>0}^{1/n} e_{\mathbb{C}}^{\sigma n}\,\underbrace{\alpha^{\mu}_{n}e_{\mathbb{C}}^{-\tau n} - \underset{\sim}{\alpha}{}^{\mu}_{-n}e_{\mathbb{C}}^{\tau n}} + e_{\mathbb{C}}^{-\sigma n}\,\underbrace{\underset{\sim}{\alpha}{}^{\mu}_{n}e_{\mathbb{C}}^{-\tau n} - \alpha^{\mu}_{-n}e_{\mathbb{C}}^{\tau n}} = \sum_{n>0}^{1/n} e_{\mathbb{C}}^{\sigma n}\,\underbrace{\alpha^{\mu}_{n}e_{\mathbb{C}}^{-\tau n} - \overset{*}{\underset{\sim}{\alpha}}{}^{\mu}_{n}e_{\mathbb{C}}^{\tau n}} + e_{\mathbb{C}}^{-\sigma n}\,\underbrace{\underset{\sim}{\alpha}{}^{\mu}_{n}e_{\mathbb{C}}^{-\tau n} - \overset{*}{\alpha}{}^{\mu}_{n}e_{\mathbb{C}}^{\tau n}}$$

$$= \sum_{n>0}^{1/n} e_{\mathbb{C}}^{\sigma n}\,\underbrace{X^{\mu}_{n}e_{\mathbb{C}}^{-\tau n} - \overset{*}{X}{}^{\mu}_{-n}e_{\mathbb{C}}^{\tau n}} + e_{\mathbb{C}}^{-\sigma n}\,\underbrace{X^{\mu}_{-n}e_{\mathbb{C}}^{-\tau n} - \overset{*}{X}{}^{\mu}_{n}e_{\mathbb{C}}^{\tau n}} = \sum_{n\neq 0}^{1/\overline{n}} e_{\mathbb{C}}^{\sigma n}\,\underbrace{X^{\mu}_{n}e_{\mathbb{C}}^{-\tau\overline{n}} - \overset{*}{X}{}^{\mu}_{-n}e_{\mathbb{C}}^{\tau\overline{n}}}$$

$${}^{\sigma}_{0}(\tau)^{\mu} - \frac{\alpha'}{\not\ell}\,{}_{0}()^{\mu} = \frac{\sqrt{\alpha'/2}}{\not\ell}\sum_{n\neq 0}^{\frac{1/2}{\overline{n}}} e_{\mathbb{C}}^{\sigma n}\,\underbrace{()^{\mu}_{n}e_{\mathbb{C}}^{-\tau\overline{n}} + \overset{*}{()}{}^{\mu}_{-n}e_{\mathbb{C}}^{\tau\overline{n}}} = \frac{\sqrt{\alpha'/2}}{\not\ell}\sum_{n\neq 0}^{\frac{1/2}{\overline{n}}}\underbrace{()^{\mu}_{n}e_{\mathbb{C}}^{\sigma n\,-\,\tau\overline{n}} + \overset{*}{()}{}^{\mu}_{n}e_{\mathbb{C}}^{\tau\overline{n}\,-\,\sigma n}}$$

$${}^{\sigma}_{0}(\tau)^{\mu} = \frac{\alpha'}{\not\ell}\,{}_{0}()^{\mu} + i\sqrt{\alpha'/2}\sum_{n\neq 0}^{\overline{n}^{-\,1/2}} e_{\mathbb{C}}^{\sigma n}\,\underbrace{-\frac{i\overline{n}}{\not\ell}\,()^{\mu}_{n}e_{\mathbb{C}}^{-\tau\overline{n}} - \frac{i\overline{n}}{\not\ell}\,\overset{*}{()}{}^{\mu}_{-n}e_{\mathbb{C}}^{\tau\overline{n}}}$$

$$()^{\mu} = \int_{d\sigma/\ell}^{0|\ell} {}^{\sigma}(\tau)^{\mu} - \tau\,{}^{\sigma}_{0}(\tau)^{\mu} : \quad {}_{0}()^{\mu} = \frac{\not\ell}{\alpha'} \int_{d\sigma/\ell}^{0|\ell} {}^{\sigma}_{0}(\tau)^{\mu}$$

$$()^{\mu} + \frac{\alpha'}{\not\ell}\,{}_{0}()^{\mu}\,\tau = \int_{d\sigma/\ell}^{0|\ell} {}^{\sigma}(\tau)^{\mu} : \quad \frac{\alpha'}{\not\ell}\,{}_{0}()^{\mu} = \int_{d\sigma/\ell}^{0|\ell} {}^{\sigma}_{0}(\tau)^{\mu}$$

$$()^{\mu} \times {}_{0}()^{\nu} = i\,\eta^{\mu\nu}$$

$$\alpha'\,()^{\mu} \times {}_{0}()^{\nu} = \int_{d\varrho/\ell}^{0|\ell} \underbrace{{}^{\varrho}(\tau)^{\mu} - \tau\,{}^{\varrho}_{0}(\tau)^{\mu}} \times \int_{d\sigma/\ell}^{0|\ell} \not\ell\,{}^{\sigma}_{0}(\tau)^{\nu}$$

$$= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \underbrace{\not\ell\,{}^{\varrho}(\tau)^{\mu} \times {}^{\sigma}_{0}(\tau)^{\nu}}_{=\ell\alpha' i\eta^{\mu\nu}\delta^{\varrho\sigma}} - \not\ell\,\tau \underbrace{{}^{\varrho}_{0}(\tau)^{\mu} \times {}^{\sigma}_{0}(\tau)^{\nu}}_{=0} = \alpha'\,i\,\eta^{\mu\nu} \int_{d\sigma/\ell}^{0|\ell} = \alpha'\,i\,\eta^{\mu\nu}$$

$$\sqrt{2\alpha'}\,()^{\mu}_{n} = e_{\mathbb{C}}^{\tau\overline{n}} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n} \underbrace{-i\,\overline{n}^{1/2\,\sigma}(\tau)^{\mu} + \not{\ell}^{\sigma}_{0}(\tau)^{\mu}/\,\overline{n}^{1/2}}$$

$$\sqrt{2\alpha'}\,\overset{*}{()}^{\mu}_{n} = e_{\mathbb{C}}^{-\tau\overline{n}} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{\sigma n} \underbrace{i\,\overline{n}^{1/2\,\sigma}(\tau)^{\mu} + \not{\ell}^{\sigma}_{0}(\tau)^{\mu}/\,\overline{n}^{1/2}}$$

$$()^{\mu}_{n}\, e_{\mathbb{C}}^{-\tau\overline{n}} - \overset{*}{()}^{\mu}_{-n}\, e_{\mathbb{C}}^{\tau\overline{n}} = \frac{-i}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n}\, {}^{\sigma}(\tau)^{\mu}\ \overline{n}^{1/2}$$

$$()^{\mu}_{n}\, e_{\mathbb{C}}^{-\tau\overline{n}} + \overset{*}{()}^{\mu}_{-n}\, e_{\mathbb{C}}^{\tau\overline{n}} = \frac{\not{\ell}}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n}\, {}^{\sigma}_{0}(\tau)^{\mu}/\ \overline{n}^{1/2}$$

$$2\,()^{\mu}_{n}\, e_{\mathbb{C}}^{-\tau\overline{n}} = \frac{-i}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n}\, {}^{\sigma}(\tau)^{\mu}\ \overline{n}^{1/2} + \frac{\not{\ell}}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n}\, {}^{\sigma}_{0}(\tau)^{\mu}/\ \overline{n}^{1/2}$$

$$2\,\overset{*}{()}^{\mu}_{-n}\, e_{\mathbb{C}}^{\tau\overline{n}} = \frac{i}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n}\, {}^{\sigma}(\tau)^{\mu}\ \overline{n}^{1/2} + \frac{\not{\ell}}{\sqrt{\alpha'/2}} \int_{d\sigma}^{0|\ell} e_{\mathbb{C}}^{-\sigma n}\, {}^{\sigma}_{0}(\tau)^{\mu}/\ \overline{n}^{1/2}$$

$$()^{\mu} \times ()_{n}^{\nu} = 0: \quad {}_{0}()^{\mu} \times ()_{n}^{\nu} = 0$$

$$()^{\mu} \times ()_{n}^{\nu} = \int_{d\varrho/\ell}^{0|\ell} \underbrace{{}^{\varrho}(\tau)^{\mu} - \tau\, {}_{0}^{\varrho}(\tau)^{\mu}} \times \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n} \underbrace{-i^{\sigma}(\tau)^{\nu}\, \overline{n}^{1/2} + \ell\!\!\!/\, {}_{0}^{\sigma}(\tau)^{\nu} / \overline{n}^{1/2}}$$

$$= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n} \underbrace{{}^{\varrho}(\tau)^{\mu} - \tau\, {}_{0}^{\varrho}(\tau)^{\mu}} \times \underbrace{-i^{\sigma}(\tau)^{\nu}\, \overline{n}^{1/2} + \ell\!\!\!/\, {}_{0}^{\sigma}(\tau)^{\nu} / \overline{n}^{1/2}}$$

$$= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n} \overbrace{-i \underbrace{{}^{\varrho}(\tau)^{\mu} \times {}^{\sigma}(\tau)^{\nu}}_{=0} \overline{n}^{1/2} + \ell\!\!\!/ \underbrace{{}^{\varrho}(\tau)^{\mu} \times {}_{0}^{\sigma}(\tau)^{\nu}}_{=\ell i \alpha' \eta^{\mu\nu} \delta^{\varrho\sigma}} / \overline{n}^{1/2} + i\tau \underbrace{{}_{0}^{\varrho}(\tau)^{\mu} \times {}^{\sigma}(\tau)^{\nu}}_{=-2\pi i \alpha' \eta^{\mu\nu} \delta^{\varrho\sigma}} \overline{n}^{1/2} - \ell\!\!\!/ \tau \underbrace{{}_{0}^{\varrho}(\tau)^{\mu} \times {}_{0}^{\sigma}(\tau)^{\nu}}_{=0} / \overline{n}^{1/2}}$$

$$= \eta^{\mu\nu}\, \alpha' \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n} \underbrace{i / \overline{n}^{1/2} - \tau \frac{2\pi}{\ell} \overline{n}^{1/2}} = 0$$

$$()_{m}^{\mu} \times ()_{n}^{\nu} = 0$$

$$2\,\alpha'\, e_{\mathbb{C}}^{-\tau(\overline{m} + \overline{n})}\, ()_{m}^{\mu} \times ()_{n}^{\nu} = \int_{d\varrho/\ell}^{0|\ell} e_{\mathbb{C}}^{-\varrho m} \underbrace{-i^{\varrho}(\tau)^{\mu}\, \overline{m}^{1/2} + \ell\!\!\!/\, {}_{0}^{\varrho}(\tau)^{\mu} / \overline{m}^{1/2}} \times \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma n} \underbrace{-i^{\sigma}(\tau)^{\nu}\, \overline{n}^{1/2} + \ell\!\!\!/\, {}_{0}^{\sigma}(\tau)^{\nu} / \overline{n}^{1/2}}$$

$$= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\varrho m}\, e_{\mathbb{C}}^{-\sigma n} \underbrace{-i^{\varrho}(\tau)^{\mu}\, \overline{m}^{1/2} + \ell\!\!\!/\, {}_{0}^{\varrho}(\tau)^{\mu} / \overline{m}^{1/2}} \times \underbrace{-i^{\sigma}(\tau)^{\nu}\, \overline{n}^{1/2} + \ell\!\!\!/\, {}_{0}^{\sigma}(\tau)^{\nu} / \overline{n}^{1/2}}$$

$$= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\varrho m}\, e_{\mathbb{C}}^{-\sigma n} \overbrace{- \underbrace{{}^{\varrho}(\tau)^{\mu} \times {}^{\sigma}(\tau)^{\nu}}_{=0} \overline{mn}^{1/2} - \ell\!\!\!/ i \underbrace{{}_{0}^{\varrho}(\tau)^{\mu} \times {}^{\sigma}(\tau)^{\nu}}_{=\ell\alpha' \eta^{\mu\nu} \delta^{\varrho\sigma}} \overline{n/m}^{1/2} - \ell\!\!\!/ i \underbrace{{}^{\varrho}(\tau)^{\mu} \times {}_{0}^{\sigma}(\tau)^{\nu}}_{-\ell\alpha' \eta^{\nu\mu} \delta^{\sigma\varrho}} \overline{m/n}^{1/2} + \ell\!\!\!/^{2} \underbrace{{}_{0}^{\varrho}(\tau)^{\mu} \times {}_{0}^{\sigma}(\tau)^{\nu}}_{=0} / \overline{mn}^{1/2}}$$

$$= \alpha'\, \eta^{\mu\nu} \underbrace{\overline{m/n}^{1/2} - \overline{n/m}^{1/2}} \underbrace{\int_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\sigma(m+n)}}_{\delta_{m+n}} = \alpha'\, \eta^{\mu\nu} \underbrace{\overline{m/n}^{1/2} - \overline{n/m}^{1/2}}\, \delta_{m+n} = 0$$

$$\left(\right)^{\mu}_{m} \times \overset{*}{\left(\right)}^{\nu}_{n} = \overset{\mu\nu}{\eta}\, \delta_{mn}$$

$$2\,\alpha'\, e_{\mathbb{C}}^{\tau\,(\overline{n} - \overline{m})} \left(\right)^{\mu}_{m} \times \overset{*}{\left(\right)}^{\nu}_{n} = \int\limits_{d\varrho/\ell}^{0|\ell} e_{\mathbb{C}}^{-\varrho m} \underbrace{-\overset{\varrho}{i}(\tau)^{\mu}\overset{1/2}{\overline{m}} + \underset{0}{\overset{\varrho}{\not\ell}}(\tau)^{\mu}/\overset{1/2}{\overline{m}}} \times \int\limits_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{\sigma n} \underbrace{\overset{\sigma}{i}(\tau)^{\nu}\,\overset{1/2}{\overline{n}} + \underset{0}{\overset{\sigma}{\not\ell}}(\tau)^{\nu}/\,\overset{1/2}{\overline{n}}}$$

$$= \int\limits_{d\varrho/\ell}^{0|\ell}\int\limits_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\varrho m}\, e_{\mathbb{C}}^{\sigma n} \underbrace{-\overset{\varrho}{i}(\tau)^{\mu}\overset{1/2}{\overline{m}} + \underset{0}{\overset{\varrho}{\not\ell}}(\tau)^{\mu}/\overset{1/2}{\overline{m}}} \times \underbrace{\overset{\sigma}{i}(\tau)^{\nu}\,\overset{1/2}{\overline{n}} + \underset{0}{\overset{\sigma}{\not\ell}}(\tau)^{\nu}/\,\overset{1/2}{\overline{n}}}$$

$$= \int\limits_{d\varrho/\ell}^{0|\ell}\int\limits_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{-\varrho m}\, e_{\mathbb{C}}^{\sigma n} \overbrace{\underbrace{{}^{\varrho}(\tau)^{\mu} \times {}^{\sigma}(\tau)^{\nu}}_{=0}\overset{1/2}{\overline{mn}} + \not\ell\underbrace{\underset{0}{\overset{\varrho}{i}}(\tau)^{\mu} \times {}^{\sigma}(\tau)^{\nu}}_{= -\ell\alpha'\overset{\mu\nu}{\eta}\overset{\varrho\sigma}{\delta}}\overset{1/2}{\overline{n/m}} - \not\ell\underbrace{\overset{\varrho}{i}(\tau)^{\mu} \times \underset{0}{\overset{\sigma}{}}(\tau)^{\nu}}_{\ell\alpha'\overset{\nu\mu}{\eta}\overset{\sigma\varrho}{\delta}}\overset{1/2}{\overline{m/n}} + \not\ell^{2}\underbrace{\underset{0}{\overset{\varrho}{}}(\tau)^{\mu} \times \underset{0}{\overset{\sigma}{}}(\tau)^{\nu}}_{=0}/\overset{1/2}{\overline{mn}}}$$

$$= \alpha'\,\overset{\mu\nu}{\eta}\,\underbrace{\overset{1/2}{\overline{n/m}} + \overset{1/2}{\overline{m/n}}}\,\underbrace{\int\limits_{d\sigma/\ell}^{0|\ell} e_{\mathbb{C}}^{\sigma\,(n - m)}}_{\delta_{mn}} = 2\,\alpha'\,\overset{\mu\nu}{\eta}\,\delta_{mn}$$