$$-\pi|\pi_{\underset{\infty}{\triangle}}\mathbb{R}^{1:d} \ni \begin{cases} {}^{2\sigma n}\mathfrak{c} \\ {}^{2\sigma n}\mathfrak{s} \end{cases}$$

$$z = {}^{2i\,(\tau+\sigma)}\mathfrak{e} \text{ left-moving=holomorphic}$$

$$\bar{z} = {}^{2i\,(\tau-\sigma)}\mathfrak{e} \text{ right-moving=anti-holomorphic}$$

$$\tau \in \mathbb{R}i \text{ imaginary time}$$

$$ {}^{z}X^{\mu} = \sum_{n}^{\mathbb{Z}} \bigotimes_{m}^{<n} \jmath_m \text{⧗} {}^{z}X_n^{\mu} \text{⧗} \bigotimes_{m}^{>n} \jmath_m$$

$$\begin{cases} {}^{z}X_0^{\mu} = X^{\mu} - {}^{z}\not{\mathfrak{e}}\, iP^{\mu} & n = 0 \\ {}^{z}X_n^{\mu} = \dfrac{\overset{*}{\partial}{}^{\mu} z^{n} - \partial^{\mu} z^{-n}}{2i\sqrt{n}} & n > 0 \\ {}^{z}X_{-n}^{\mu} = \dfrac{\overset{*}{\partial}{}^{\mu} \bar{z}^{n} - \partial^{\mu} \bar{z}^{-n}}{2i\sqrt{n}} & n > 0 \end{cases}$$

$$ {}^{\sigma\tau}X^{\mu} = {}^{\sigma+\pi:\tau}X^{\mu} \text{ bound cond}$$

$$ {}^{\sigma\tau}X^{\mu} = {}^{\tau-\sigma}X_R^{\mu} + {}^{\tau+\sigma}X_L^{\mu}$$

$$= \frac{X^{\mu} + P^{\mu}(\tau-\sigma)}{2} - \sum_{n>0} \alpha_n^{\mu} \frac{{}^{-2in\,(\tau-\sigma)}\mathfrak{e}}{2in} + \sum_{n>0} \alpha_{-n}^{\mu} \frac{{}^{2in\,(\tau-\sigma)}\mathfrak{e}}{2in}$$

$$+ \frac{X^{\mu} + P^{\mu}(\tau+\sigma)}{2} - \sum_{n>0} \utilde{\alpha}_n^{\mu} \frac{{}^{-2in\,(\tau+\sigma)}\mathfrak{e}}{2in} + \sum_{n>0} \utilde{\alpha}_{-n}^{\mu} \frac{{}^{2in\,(\tau+\sigma)}\mathfrak{e}}{2in}$$

$$= X^{\mu} + P^{\mu}\tau - \sum_{n>0} \alpha_n^{\mu} \frac{\bar{z}^{-n}}{2in} + \sum_{n>0} \overset{*}{\alpha}{}_n^{\mu} \frac{\bar{z}^{n}}{2in} - \sum_{n>0} \utilde{\alpha}_n^{\mu} \frac{z^{-n}}{2in} + \sum_{n>0} \overset{*}{\utilde{\alpha}}{}_n^{\mu} \frac{z^{n}}{2in}$$

$$= X^{\mu} + P^{\mu}\tau + \sum_{n>0} \frac{\overset{*}{\alpha}{}_n^{\mu}\bar{z}^{n} - \alpha_n^{\mu}\bar{z}^{-n}}{2in} + \sum_{n>0} \frac{\overset{*}{\utilde{\alpha}}{}_n^{\mu} z^{n} - \utilde{\alpha}_n^{\mu} z^{-n}}{2in}$$

$$\tau = 0$$

$$ {}^{\sigma}X^{\mu} = {}^{-\sigma}X_R^{\mu} + {}^{\sigma}X_L^{\mu}$$

$$= \frac{X^{\mu} - \sigma P^{\mu}}{2} - \sum_{n>0} \alpha_n^{\mu} \frac{{}^{2in\sigma}\mathfrak{e}}{2in} + \sum_{n>0} \alpha_{-n}^{\mu} \frac{{}^{-2in\sigma}\mathfrak{e}}{2in} + \frac{X^{\mu} + P^{\mu}\sigma}{2} - \sum_{n>0} \utilde{\alpha}_n^{\mu} \frac{{}^{-2in\sigma}\mathfrak{e}}{2in} + \sum_{n>0} \utilde{\alpha}_{-n}^{\mu} \frac{{}^{2in\sigma}\mathfrak{e}}{2in}$$

$$X^{\mu} \ast P^{\nu} = i\,\eta^{\mu\nu}$$

$$\underline{X}^{\mu}_{m} \ast \underline{X}^{\nu}_{n} = m\,\delta_{m+n}\,\eta^{\mu\nu} = \bar{X}^{\mu}_{m} \ast \bar{X}^{\nu}_{n}$$

equal time commutators

$${}^{\tau}\dot{X}^{\mu}_{\varrho} \ast {}^{\tau}X^{\nu}_{\sigma} = -i\delta\left(\sigma - \varrho\right)\eta^{\mu\nu}$$

$${}^{\tau}X^{\mu}_{\varrho} \ast {}^{\tau}X^{\nu}_{\sigma} = 0 = {}^{\tau}\dot{X}^{\mu}_{\varrho} \ast {}^{\tau}\dot{X}^{\nu}_{\sigma}$$

$$k \cdot X = \sum_{\mu\nu} k^{\mu}\,\eta_{\mu\nu}\,X^{\nu} = \sum_{\mu\nu} k^{\mu}\,\eta_{\mu\nu}\,X^{\nu} \bowtie \imath^{\not\nu}$$