

$$\Delta = \frac{1}{2} \overbrace{L_0 + \bar{L}_0 - 2}^{-1} = \int_{du}^{0|1} u^{L_0 + \bar{L}_0 - 3}$$

$$V = \int_{d\sigma}^{0|\pi} 2\sigma i \mathbf{e}^{L_0 - \bar{L}_0} \underbrace{V \mathbf{X} \bar{V}}^{-2\sigma i \mathbf{e}^{L_0 - \bar{L}_0}} = \int_{ds}^{0|\pi} si \mathbf{e}^{L_0 - \bar{L}_0} \underbrace{V \mathbf{X} \bar{V}}^{-si \mathbf{e}^{L_0 - \bar{L}_0}}$$

$$\int_{dz}^{\mathbb{B}} z^{L_0 - 2} \bar{z}^{\bar{L}_0 - 2} = \int_{du}^{0|1} \int_{ds}^{0|\pi} u \overbrace{u^{is} \mathbf{e}}^{L_0 - 2} \overbrace{u^{-is} \mathbf{e}}^{\bar{L}_0 - 2}$$

$$= \int_{du}^{0|1} \int_{ds}^{0|\pi} u u^{L_0 + \bar{L}_0 - 4} is \mathbf{e}^{L_0 - \bar{L}_0} = \int_{du}^{0|1} u^{L_0 + \bar{L}_0 - 3} \int_{ds}^{0|\pi} is \mathbf{e}^{L_0 - \bar{L}_0}$$

$$\Phi_1 | V_2 \Delta \cdots \Delta V_{m-1} \Phi_m = \Phi_1 \underbrace{V_{-2} \mathbf{X} \bar{V}_2}_{\mathbb{B}} \int_{dz_2} z_2^{L_0-2} \bar{z}_2^{\bar{L}_0-2} \underbrace{V_{-3} \mathbf{X} \bar{V}_3}_{\mathbb{B}} \int_{dz_3} z_3^{L_0-2} \bar{z}_3^{\bar{L}_0-2} \underbrace{V_{-4} \mathbf{X} \bar{V}_4}_{\mathbb{B}} \cdots$$

$$\int_{dz_{m-3}} z_{m-3}^{L_0-2} \bar{z}_{m-3}^{\bar{L}_0-2} \underbrace{V_{-m-2} \mathbf{X} \bar{V}_{m-2}}_{\mathbb{B}} \int_{dz_{m-2}} z_{m-2}^{L_0-2} \bar{z}_{m-2}^{\bar{L}_0-2} \underbrace{V_{-m-1} \mathbf{X} \bar{V}_{m-1}}_{\mathbb{B}} \Phi_m$$

$$\begin{aligned} \text{LHS} &= \Phi_1 \int_{ds_2}^{0|\pi} s_2^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-2} \mathbf{X} \bar{V}_2}_{\mathbb{B}}^{-s_2 i} \mathbf{e}^{L-\bar{L}} \int_{du_2}^{0|1} u_2^{L_0+\bar{L}_0-3} \\ &\int_{ds_3}^{0|\pi} s_3^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-3} \mathbf{X} \bar{V}_3}_{\mathbb{B}}^{-s_3 i} \mathbf{e}^{L-\bar{L}} \int_{du_3}^{0|1} u_3^{L_0+\bar{L}_0-3} \int_{ds_4}^{0|\pi} s_4^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-4} \mathbf{X} \bar{V}_4}_{\mathbb{B}}^{-s_4 i} \mathbf{e}^{L-\bar{L}} \int_{du_4}^{0|1} u_4^{L_0+\bar{L}_0-3} \\ &\cdots \int_{du_{m-2}}^{0|1} u_{m-2}^{L_0+\bar{L}_0-3} \int_{ds_{m-1}}^{0|\pi} s_{m-1}^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-m-1} \mathbf{X} \bar{V}_{m-1}}_{\mathbb{B}}^{-s_{m-1} i} \mathbf{e}^{L-\bar{L}} \Phi_m = \Phi_1 \int_{ds_2}^{0|\pi} \underbrace{V_{-2} \mathbf{X} \bar{V}_2}_{\mathbb{B}} \int_{du_2}^{0|1} u_2^{L_0+\bar{L}_0-3} \\ &\int_{ds_3}^{0|\pi} (s_3-s_2)^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-3} \mathbf{X} \bar{V}_3}_{\mathbb{B}} \int_{du_3}^{0|1} u_3^{L_0+\bar{L}_0-3} \int_{ds_4}^{0|\pi} (s_4-s_3)^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-4} \mathbf{X} \bar{V}_4}_{\mathbb{B}} \int_{du_4}^{0|1} u_4^{L_0+\bar{L}_0-3} \\ &\cdots \int_{ds_{m-2}}^{0|\pi} (s_{m-2}-s_{m-3})^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-m-2} \mathbf{X} \bar{V}_{m-2}}_{\mathbb{B}} \int_{du_{m-2}}^{0|1} u_{m-2}^{L_0+\bar{L}_0-3} \int_{ds_{m-1}}^{0|\pi} (s_{m-1}-s_{m-2})^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-m-1} \mathbf{X} \bar{V}_{m-1}}_{\mathbb{B}} \Phi_m \\ &\stackrel{t_k = s_{k+1}-s_k}{=} \Phi_1 \int_{ds_2}^{0|\pi} \underbrace{V_{-2} \mathbf{X} \bar{V}_2}_{\mathbb{B}} \int_{du_2}^{0|1} u_2^{L_0+\bar{L}_0-3} \int_{dt_2}^{0|\pi} t_2^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-3} \mathbf{X} \bar{V}_3}_{\mathbb{B}} \int_{du_3}^{0|1} u_3^{L_0+\bar{L}_0-3} \int_{dt_3}^{0|\pi} t_3^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-4} \mathbf{X} \bar{V}_4}_{\mathbb{B}} \cdots \int_{du_{m-3}}^{0|1} u_{m-3}^{L_0+\bar{L}_0-3} \\ &\int_{dt_{m-3}}^{0|\pi} t_{m-3}^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-m-2} \mathbf{X} \bar{V}_{m-2}}_{\mathbb{B}} \int_{du_{m-2}}^{0|1} u_{m-2}^{L_0+\bar{L}_0-3} \int_{dt_{m-2}}^{0|\pi} t_{m-2}^i \mathbf{e}^{L-\bar{L}} \underbrace{V_{-m-1} \mathbf{X} \bar{V}_{m-1}}_{\mathbb{B}} \Phi_m = \text{RHS} \end{aligned}$$

$$\underbrace{w_1 V_1 \cdots w_m V_m}_{\mathbb{B}} = w_{\sigma_1} V_{\sigma_1} \cdots w_{\sigma_m} V_{\sigma_m} : \overline{w_{\sigma_1}} \geq \cdots \geq \overline{w_{\sigma_m}}$$

$$\int_{\mathbb{C}^m} dw_1 \cdots dw_m \overleftarrow{w_1 V_1 \cdots w_m V_m} = \sum_{\sigma}^{m!} \int_{dw_1 \cdots dw_m}^{\overline{w_1} \geq \cdots \geq \overline{w_m}} w_1 V_{\sigma_1} \cdots w_m V_{\sigma_m}$$

$$\text{LHS} = \sum_{\sigma}^{m!} \int_{dw_1 \cdots dw_m}^{\overline{w_{\sigma_1}} \geq \cdots \geq \overline{w_{\sigma_m}}} \overleftarrow{w_1 V_1 \cdots w_m V_m} = \sum_{\sigma}^{m!} \int_{dw_1 \cdots dw_m}^{\overline{w_{\sigma_1}} \geq \cdots \geq \overline{w_{\sigma_m}}} w_{\sigma_1} V_{\sigma_1} \cdots w_{\sigma_m} V_{\sigma_m} = \text{RHS}$$

$$\begin{aligned} & \left\{ \begin{array}{l} \Phi_1 \overset{\infty}{\sim} \overset{w_1}{\sim} |\Omega| w_1^{w_1} V_1 \\ \frac{w_m V_m}{w_m} \Omega \overset{w_m}{\sim} \Phi_m \end{array} \right\} \left\{ \begin{array}{l} \Phi_1 \overset{\infty}{\sim} \overset{\bar{w}_1}{\sim} |\Omega| \bar{w}_1^{\bar{w}_1} \bar{V}_1 \\ \frac{\bar{w}_m \bar{V}_m}{\bar{w}_m} \Omega \overset{\bar{w}_m}{\sim} \Phi_m \end{array} \right\} \\ \Rightarrow & \Lambda^{w_1 1 w_m} \overline{V_1 \cdots V_m} \Lambda^{\bar{w}_1 1 \bar{w}_m} \overline{V_1 \cdots V_m} \overset{\infty}{\sim} \overset{w_1}{\sim} \Phi_1 |V_2 \otimes \bar{V}_2 \Delta V_3 \otimes \bar{V}_3 \Delta \cdots \Delta V_{m-1} \otimes \bar{V}_{m-1} \Phi_m \end{aligned}$$

$$\infty > \overline{w_1} > \overline{w_2} = 1 > \overline{w_3} > \cdots > \overline{w_{m-1}} > \overline{w_m} > 0: \quad 3 \leq i \leq m-1: \quad z_i = w_i / w_{i-1} \in \mathbb{B}^\times$$

$$\Lambda^{w_1 1 w_m} \overline{V_1 \cdots V_m} \Lambda^{\bar{w}_1 1 \bar{w}_m} \overline{V_1 \cdots V_m} = \overline{w_1 - 1} \overline{w_1 - w_m} \overline{1 - w_m} \int_{dw_3 \cdots dw_{m-1}}^{1 > \overline{w_3} > \cdots > \overline{w_{m-1}} > 0}$$

$$\Omega \left| \frac{w_1 V_1 V_2}{w_1} \frac{w_3 V_3}{1} \cdots \frac{w_{m-1} V_{m-1}}{w_{m-1}} \frac{w_m V_m}{w_m} \Omega \bar{\Omega} \right| \frac{\bar{w}_1 \bar{V}_1 \bar{V}_2}{\bar{w}_1} \frac{\bar{w}_3 \bar{V}_3}{\bar{w}_3} \cdots \frac{\bar{w}_{m-1} \bar{V}_{m-1}}{\bar{w}_{m-1}} \frac{\bar{w}_m \bar{V}_m}{\bar{w}_m} \bar{\Omega} = \overline{1 - 1/w_1} \overline{1 - w_m/w_1} \overline{1 - w_m} \underbrace{\Omega |w_1^{w_1} V_1 \bar{\Omega}|}_{\sim \Phi_1} \underbrace{|\bar{w}_1^{\bar{w}_1} \bar{V}_1 \bar{\Omega}|}_{\sim \bar{\Phi}_1}$$

$$\int_{dw_3 \cdots dw_{m-1}}^{1 > \overline{w_3} > \cdots > \overline{w_{m-1}} > 0} V_2 \otimes \bar{V}_2 \frac{w_3 V_3}{w_3} \otimes \bar{V}_3 \cdots \frac{w_{m-1} V_{m-1}}{w_{m-1}} \otimes \bar{V}_{m-1} \underbrace{w_m V_m \Omega / w_m}_{\sim \Phi_m} \underbrace{\bar{w}_m \bar{V}_m \bar{\Omega} / \bar{w}_m}_{\sim \bar{\Phi}_m}$$

$$\sim \int_{dw_3 \cdots dw_{m-1}}^{1 > \overline{w_3} > \cdots > \overline{w_{m-1}} > 0} \Phi_1 |V_2 \otimes \bar{V}_2 \frac{w_3 V_3}{w_3} \otimes \bar{V}_3 \cdots \frac{w_{m-1} V_{m-1}}{w_{m-1}} \otimes \bar{V}_{m-1} \Phi_m$$

$$= \int_{dz_3}^{\mathbb{B}} \cdots \int_{dz_{m-1}}^{\mathbb{B}} \Phi_1 |V_2 \otimes \bar{V}_2 \frac{z_3 V_3}{z_3} \otimes \bar{V}_3 \frac{z_3 z_4 V_4}{z_3} \otimes \bar{V}_4 \cdots \frac{z_{m-1} V_{m-1}}{z_{m-1}} \otimes \bar{V}_{m-1} \Phi_m$$

$$= \int_{dz_3}^{\mathbb{B}} \cdots \int_{dz_{m-1}}^{\mathbb{B}} \Phi_1 |V_2 \otimes \bar{V}_2 z_3^L \otimes \bar{z}_3^{\bar{L}} \frac{V_3}{z_3} \otimes \bar{\frac{V_3}{z_3}} z_3^{-L} \otimes \bar{z}_3^{-\bar{L}} \overbrace{z_3 z_4}^L \otimes \overbrace{z_3 z_4}^{\bar{L}} \frac{V_4}{z_4} \otimes \bar{\frac{V_4}{z_4}} \overbrace{z_3 z_4}^{-L} \otimes \overbrace{z_3 z_4}^{-\bar{L}}$$

$$\cdots \overbrace{z_3 \cdots z_{m-1}}^L \otimes \overbrace{z_3 \cdots z_{m-1}}^{\bar{L}} \frac{V_{m-1}}{z_{m-1}} \otimes \bar{\frac{V_{m-1}}{z_{m-1}}} \overbrace{z_3 \cdots z_{m-1}}^{-L} \otimes \overbrace{z_3 \cdots z_{m-1}}^{-\bar{L}} \bar{\Phi}_m$$

$$= \int_{dz_3}^{\mathbb{B}} \cdots \int_{dz_{m-1}}^{\mathbb{B}} \Phi_1 |V_2 \otimes \bar{V}_2 z_3^L \otimes \bar{z}_3^{\bar{L}} \frac{V_3}{z_3} \otimes \bar{\frac{V_3}{z_3}} z_4^L \otimes \bar{z}_4^{\bar{L}} \frac{V_4}{z_4} \otimes \bar{\frac{V_4}{z_4}} \cdots z_{m-1}^L \otimes \bar{z}_{m-1}^{\bar{L}} \frac{V_{m-1}}{z_{m-1}} \otimes \bar{\frac{V_{m-1}}{z_{m-1}}} \overbrace{z_3 \cdots z_{m-1}}^{-1} \otimes \overbrace{z_3 \cdots z_{m-1}}^{-\bar{1}} \Phi_m$$

$$= \int_{dz_3}^{\mathbb{B}} \cdots \int_{dz_{m-1}}^{\mathbb{B}} \Phi_1 |V_2 \otimes \bar{V}_2 z_3^{L-2} \otimes \bar{z}_3^{\bar{L}-2} V_3 \otimes \bar{V}_3 z_4^{L-2} \otimes \bar{z}_4^{\bar{L}-2} V_4 \otimes \bar{V}_4 \cdots z_{m-1}^{L-2} \otimes \bar{z}_{m-1}^{\bar{L}-2} V_{m-1} \otimes \bar{V}_{m-1} \Phi_m = \text{RHS}$$

$$\overline{z} \leq 1 \leq \overline{w}$$

$$\Omega_1 \star \underbrace{V \Delta_3 V}_2 \Omega_4 = dzd\bar{z} \Omega_1 \star_2 V \frac{z\bar{z}}{z\bar{z}} V \Omega_4 + dwd\bar{w} \Omega_1 \star \frac{w\bar{w}}{w\bar{w}} \star_2 V \Omega_4$$

$$\begin{aligned} \text{RHS} &= \frac{dzd\bar{z}}{z\bar{z}} \Omega_1 \star_2 V \frac{z\bar{z}}{z\bar{z}} V \Omega_4 + \frac{dwd\bar{w}}{w\bar{w}} \Omega_1 \star \frac{w\bar{w}}{w\bar{w}} \star_3 V \star_2 V \Omega_4 \\ &= \frac{dzd\bar{z}}{z\bar{z}} \Omega_1 \star_2 V z^L \bar{z}^{\bar{L}} \star_3 V z^{-L} \bar{z}^{-\bar{L}} \Omega_4 + \frac{dwd\bar{w}}{w\bar{w}} \Omega_1 \star w^L \bar{w}^{\bar{L}} \star_3 V w^{-L} \bar{w}^{-\bar{L}} \star_2 V \Omega_4 \\ &= \frac{dzd\bar{z}}{z\bar{z}} \Omega_1 \star_2 V z^L \bar{z}^{\bar{L}} \star_3 V z^{-1} \bar{z}^{-1} \Omega_4 + \frac{dwd\bar{w}}{w\bar{w}} \Omega_1 \star w \bar{w} \star_3 V w^{-L} \bar{w}^{-\bar{L}} \star_2 V \Omega_4 \\ &= \frac{dzd\bar{z}}{z\bar{z}} \Omega_1 \star_2 V z^L \bar{z}^{\bar{L}} \star_3 V z^{-1} \bar{z}^{-1} \Omega_4 + \frac{dzd\bar{z}}{z\bar{z}} \Omega_1 \star z^{-1} \bar{z}^{-1} \star_3 V z^L \bar{z}^{\bar{L}} \star_2 V \Omega_4 \\ &= \frac{dzd\bar{z}}{z\bar{z}} \Omega_1 \star_2 V z^{L-1} \bar{z}^{\bar{L}-1} \star_3 V \Omega_4 + \frac{dzd\bar{z}}{z\bar{z}} \Omega_1 \star_3 V z^{L-1} \bar{z}^{\bar{L}-1} \star_2 V \Omega_4 \\ &= \Omega_1 \star_2 V \Delta_3 V \Omega_4 + \Omega_1 \star_3 V \Delta_2 V \Omega_4 = \text{LHS} \end{aligned}$$

$$\Omega \star \underbrace{V \Delta_2 V \Delta \cdots \Delta_m V}_1 \Omega = dz^1 d\bar{z}^1 \cdots dz^i d\bar{z}^i \cdots dz^j d\bar{z}^j \cdots dz^k d\bar{z}^k \cdots dz^m d\bar{z}^m \overbrace{z^i - z^j}^2 \overbrace{z^i - z^k}^2 \overbrace{z^j - z^k}^2$$

$$\Omega \star \underbrace{\frac{z^1 \bar{z}^1}{z^1 \bar{z}^1} V \cdots \frac{z^i \bar{z}^i}{z^i \bar{z}^i} V \cdots \frac{z^j \bar{z}^j}{z^j \bar{z}^j} V \cdots \frac{z^k \bar{z}^k}{z^k \bar{z}^k} V \cdots \frac{z^m \bar{z}^m}{z^m \bar{z}^m} V}_\Omega \Omega$$

$$\overline{Z-1} \Omega_1 = 0 = \overline{Z-1} \Omega_m$$

$$\Omega_1 = \lim_{y_1 \rightarrow \infty} y_1^{y_1} V_1 \Omega$$

$$\Omega_m = \lim_{y_m \rightarrow 0} \bar{y}_m^{-1} y_m^{y_m} V_m \Omega$$

$$\Omega_1 \star V_2 \overline{Z-1} V_2 \overline{Z-1} \cdots \overline{Z-1} V_{m-1} \Omega_m$$

$$= \Omega_1 \star V_2 \int_{dz_3}^{0|1} z_3^{Z-1} V_3 \int_{dz_4}^{0|1} z_4^{Z-1} V_4 \cdots \int_{dz_{m-1}}^{0|1} z_{m-1}^{Z-1} V_{m-1} \Omega_m$$

$$\begin{aligned}
&= \int_{dz_3}^{0|1} \int_{dz_4}^{0|1} \int_{dz_{m-1}}^{0|1} \Omega_1 \star V_2 z_3^{Z-1} V_3 z_4^{Z-1} V_4 \cdots z_{m-1}^{Z-1} V_{m-1} \Omega_m \\
&= \int_{dz_3/z_3}^{0|1} \int_{dz_4/z_4}^{0|1} \int_{dz_{m-1}/z_{m-1}}^{0|1} \Omega_1 \star^1 V_2 z_3 V_3 z_3 z_4 V_4 \cdots z_3 z_4 \cdots z_{m-1} V_{m-1} \Omega_m \\
&= \int_{dy_i/y_i}^{1=y_2 \geq y_3 \geq y_4 \geq \cdots \geq y_{m-1}} \Omega_1 \star^{y_2} V_2^{y_3} V_3^{y_4} V_4 \cdots^{y_{m-1}} V_{m-1} \Omega_m
\end{aligned}$$

$$\Omega \star \frac{z_1 V}{z_1} \cdots \frac{z_m V}{z_m} \Omega$$

Fadeev-Popov gauge fixing :  $1 \leq i < j < k \leq m$ :  $\infty \geq a > b > c \geq 0$

$$\begin{aligned}
&d\mu_{i:j:k}^{a:b:c} (z_1 \cdots z_{m-3}) \\
&= \frac{2}{a-b} \frac{2}{a-c} \frac{2}{b-c} \int_{dz_1 d\bar{z}_1 \cdots dz_{m-3} d\bar{z}_{m-3}} \\
&d\mu_{1:2:m}^{\infty:1:0} (z_1 \cdots z_{m-3}) = \frac{2}{a-b} \frac{2}{a-c} \frac{2}{b-c} \int_{dz_1 d\bar{z}_1 \cdots dz_{m-3} d\bar{z}_{m-3}}
\end{aligned}$$