

$$\begin{aligned} \overline{i k \cdot P} \times \overline{k \cdot X} &= k \cdot k \\ \overline{i k^\mu \eta_{\mu\nu} P^\nu} \times \overline{k^\alpha \eta_{\alpha\beta} X^\beta} &= k^\mu \eta_{\mu\nu} k^\nu \end{aligned}$$

$$i P^\mu \times X^\nu = \overset{\mu\nu}{\eta}$$

$$\text{LHS} = k^\mu \eta_{\mu\nu} k^\alpha \eta_{\alpha\beta} i P^\nu \times X^\beta = k^\mu \eta_{\mu\nu} k^\alpha \eta_{\alpha\beta} \overset{\nu\beta}{\eta} = k^\mu \eta_{\mu\nu} k^\alpha \overset{\nu}{\delta} = \text{RHS}$$

$${}^{k \cdot X} \mathbf{e} = \exp k^\mu \eta_{\mu\nu} X^\nu i$$

$${}^x k = \exp k^\mu \eta_{\mu\nu} \underbrace{X^\nu i + P^{\nu x} \not{x}} = \begin{cases} \mathbf{e}^{k \cdot X} x^{k \cdot (P + k/2)} \\ x^{k \cdot (P - k/2)} \mathbf{e}^{k \cdot X} \end{cases}$$

$$A \times B \text{ scalar} \Rightarrow A + B \mathbf{e} = A \mathbf{e} B \mathbf{e}^{-A \times B/2} \mathbf{e} \Rightarrow -B \mathbf{e}^{-A} \mathbf{e}^{A+B} \mathbf{e} = -A \times B/2 \mathbf{e}$$

$$x^{-k \cdot P} \mathbf{e}^{-k \cdot X} \text{ LHS} = \mathbf{e}^{-k \cdot P \not{x}} \mathbf{e}^{-k \cdot X} \mathbf{e}^{k \cdot (X + P \not{x})} = \mathbf{e}^{-\overline{i k \cdot X} \times \overline{k \cdot P} \not{x}/2} = \mathbf{e}^{k \cdot k \not{x}/2} = \sqrt{x}^{k^2}$$

$$\mathbf{e}^{-k \cdot X} x^{-k \cdot P} \text{ LHS} = \mathbf{e}^{-k \cdot P \not{x}} \mathbf{e}^{-k \cdot X} \mathbf{e}^{k \cdot (X + P \not{x})} = \mathbf{e}^{-\overline{i k \cdot P} \times \overline{k \cdot X} \not{x}/2} = \mathbf{e}^{-k \cdot k \not{x}/2} = \sqrt{x}^{-k^2}$$

$$X_i \mathbf{e}^k \sqrt{x}^{P^2} = \sqrt{x}^{\overline{P^2 - k^2}} X_i \mathbf{e}^k$$

$$\sqrt{x}^{P^2} X_i \mathbf{e}^k \sqrt{x}^{-P^2} = k \cdot (X + P \not{x}) \mathbf{e} = X_i \mathbf{e}^k k \cdot P \not{x} \mathbf{e}^{-\overline{i k \cdot X} \times \overline{k \cdot P} \not{x}/2} \mathbf{e} = X_i \mathbf{e}^k k \cdot P \not{x} \mathbf{e}^{k \cdot k \not{x}/2} \mathbf{e} = X_i \mathbf{e}^k x^{k \cdot P} \sqrt{x}^{k^2}$$

$$\sqrt{x}^{P^2} X_i \mathbf{e}^k = X_i \mathbf{e}^k x^{k \cdot P} \sqrt{x}^{k^2} \sqrt{x}^{P^2} = X_i \mathbf{e}^k \sqrt{x}^{(k^2 + P^2)}$$

$$\sqrt{x}^{P^2} X_i \mathbf{e}^{-k} = X_i \mathbf{e}^{-k} \sqrt{x}^{\overline{P^2 - k^2}}$$

$${}_{x_1}V {}_{x_1x_2}V \cdots {}_{x_1x_2 \cdots x_m}V \sqrt{x_1x_2 \cdots x_m}^{P^2} = \sqrt{x_1} {}_1V \sqrt{x_2} {}_2V \cdots \sqrt{x_m} {}_mV$$

$$\text{LHS} = \sqrt{x_1} {}_1V \sqrt{x_1}^{-P^2} \sqrt{x_1x_2} {}_2V \sqrt{x_1x_2}^{-P^2} \cdots \sqrt{x_1x_2 \cdots x_m} {}_mV \sqrt{x_1x_2 \cdots x_m}^{-P^2} \sqrt{x_1x_2 \cdots x_m}^{P^2} = \text{RHS}$$

$$\sqrt{x_1}^{P^2} X_i \mathbf{e}^{k_1} \sqrt{x_2}^{P^2} X_i \mathbf{e}^{k_2} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} = \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \sqrt{x_3}^{\overbrace{2}^{P-k_1-k_2}} \cdots \sqrt{x_m}^{\overbrace{2}^{P-k_1-k_2-\cdots-k_{m-1}}}}$$

$$\begin{aligned} \sqrt{x_1}^{P^2} \underbrace{X_i \mathbf{e}^{k_1} \sqrt{x_2}^{P^2} X_i \mathbf{e}^{k_2}} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} &= \sqrt{x_1}^{P^2} \underbrace{\sqrt{x_2}^{\overbrace{2}^{P-k_1}} X_i \mathbf{e}^{k_1}} X_i \mathbf{e}^{k_2} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} = \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \underbrace{X_i \mathbf{e}^{k_1} X_i \mathbf{e}^{k_2}} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} X_i \mathbf{e}^{k_1+k_2} \sqrt{x_3}^{P^2} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} = \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \underbrace{X_i \mathbf{e}^{k_1+k_2} \sqrt{x_3}^{P^2}} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \underbrace{\sqrt{x_3}^{\overbrace{2}^{P-k_1-k_2}} X_i \mathbf{e}^{k_1+k_2} X_i \mathbf{e}^{k_3}} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \sqrt{x_3}^{\overbrace{2}^{P-k_1-k_2}} \underbrace{X_i \mathbf{e}^{k_1+k_2} X_i \mathbf{e}^{k_3}} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \sqrt{x_3}^{\overbrace{2}^{P-k_1-k_2}} X_i \mathbf{e}^{k_1+k_2+k_3} \cdots \sqrt{x_m}^{P^2} X_i \mathbf{e}^{k_m} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \cdots \sqrt{x_{m-1}}^{\overbrace{2}^{P-k_1-k_2-\cdots-k_{m-2}}} \underbrace{X_i \mathbf{e}^{k_1+k_2+\cdots+k_{m-1}} \sqrt{x_m}^{P^2}} X_i \mathbf{e}^{k_m} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \cdots \sqrt{x_{m-1}}^{\overbrace{2}^{P-k_1-k_2-\cdots-k_{m-2}}} \underbrace{\sqrt{x_m}^{\overbrace{2}^{P-k_1-\cdots-k_{m-1}-k_m}} X_i \mathbf{e}^{k_1+k_2+\cdots+k_{m-1}} X_i \mathbf{e}^{k_m}} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \cdots \sqrt{x_{m-1}}^{\overbrace{2}^{P-k_1-k_2-\cdots-k_{m-2}}} \sqrt{x_m}^{\overbrace{2}^{P-k_1-\cdots-k_{m-1}-k_m}} \underbrace{X_i \mathbf{e}^{k_1+k_2+\cdots+k_{m-1}} X_i \mathbf{e}^{k_m}} \\ &= \sqrt{x_1}^{P^2} \sqrt{x_2}^{\overbrace{2}^{P-k_1}} \cdots \sqrt{x_{m-1}}^{\overbrace{2}^{P-k_1-k_2-\cdots-k_{m-2}}} \sqrt{x_m}^{\overbrace{2}^{P-k_1-\cdots-k_{m-1}-k_m}} \underbrace{X_i \mathbf{e}^{k_1+k_2+\cdots+k_{m-1}+k_m}}_{=1} \end{aligned}$$