

$$e_{\mathbb{R}}(x) = e^{\pi i x / \ell}$$

$$0|\ell \begin{array}{c} \Delta_{\infty} \\ \tau \end{array} \mathbf{t}^{\mathbb{C}} \mathbf{z} \mathbb{R} \xrightarrow{\begin{array}{c} ()^{\mu} \\ ()^{\mu} \end{array}} \mathbb{R}: \quad 0|\ell \begin{array}{c} \Delta_{\infty} \\ \tau \end{array} \mathbf{t}^{\mathbb{C}} \mathbf{z} \mathbb{R} \xrightarrow{\begin{array}{c} ()^{\mu} \\ ()^{\mu} \end{array}} \mathbb{C}: \quad 0|\ell \begin{array}{c} \Delta_{\infty} \\ \tau \end{array} \mathbf{t}^{\mathbb{C}} \mathbf{z} \mathbb{R} \xrightarrow{\begin{array}{c} \mathbb{R}^{\mu}: ()^{\mu} \\ \mathbb{R}^{\mu}: ()^{\mu} \end{array}} \mathbb{R}$$

$$\sigma(\tau)^{\mu} = ()^{\mu} + \tau \frac{2\alpha'}{\ell} {}_0()^{\mu} + i\sqrt{2\alpha'} \sum_{n>0}^{1/\sqrt{n}} \sigma n / \ell \mathbf{c} \underbrace{()^{\mu}_{\mathbb{R}} e^{-\tau n} - {}^*()^{\mu}_{\mathbb{R}} e^{\tau n}}$$

$$\tau \sigma X^{\mu} = X^{\mu} + \tau \frac{2\alpha'}{\ell} {}_0 X^{\mu} + i\sqrt{2\alpha'} \sum_{n \neq 0}^{1/n} \sigma n / \ell \mathbf{c} \alpha_n^{\mu} e^{-\tau n}$$

$$\sum_{n \neq 0}^{1/n} \sigma n / \ell \mathbf{c} \alpha_n^{\mu} e^{-\tau n} = \sum_{n > 0}^{1/n} \sigma n / \ell \mathbf{c} \underbrace{\alpha_n^{\mu} e^{-\tau n} - \alpha_n^{\mu} e^{\tau n}}$$

$$= \sum_{n > 0}^{1/n} \sigma n / \ell \mathbf{c} \underbrace{\alpha_n^{\mu} e^{-\tau n} - {}^* \alpha_n^{\mu} e^{\tau n}} = \sum_{n > 0}^{1/\sqrt{n}} \sigma n / \ell \mathbf{c} \underbrace{()^{\mu}_{\mathbb{R}} e^{-\tau n} - {}^*()^{\mu}_{\mathbb{R}} e^{\tau n}}$$

$$\sigma_0(\tau)^{\mu} = \frac{2\alpha'}{\ell} {}_0()^{\mu} + \frac{\sqrt{\alpha'/2}}{\ell} \sum_{n>0}^{\sqrt{n}} \sigma n / \ell \mathbf{c} \underbrace{()^{\mu}_{\mathbb{R}} e^{-\tau n} + {}^*()^{\mu}_{\mathbb{R}} e^{\tau n}}$$

$$\sigma_0(\tau)^{\mu} = \frac{2\alpha'}{\ell} {}_0()^{\mu} + i\sqrt{2\alpha'} \sum_{n>0}^{1/\sqrt{n}} \sigma n / \ell \mathbf{c} \underbrace{()^{\mu}_{\mathbb{R}} e^{-\tau n} \frac{-ni}{\ell} - {}^*()^{\mu}_{\mathbb{R}} e^{\tau n} \frac{ni}{\ell}}$$

$$\sigma\gamma = \# \gamma_0 + \sqrt{2} \sum_{n>0} \sigma n / \ell \mathbf{c} \# \gamma_n \Rightarrow \begin{cases} \# \gamma_0 = \int_{d\sigma/\ell}^{0|\ell} \sigma \gamma \\ \# \gamma_m = \sqrt{2} \int_{d\sigma/\ell}^{0|\ell} \sigma m / \ell \mathbf{c} \sigma \gamma \end{cases}$$

$$m \neq n \Rightarrow 2 \int_{d\sigma/\ell}^{0|\ell} \sigma m / \ell \mathbf{c} \sigma n / \ell \mathbf{c} = \int_{d\sigma/\ell}^{0|\ell} \sigma(m+n) / \ell \mathbf{c} + \int_{d\sigma/\ell}^{0|\ell} \sigma(m-n) / \ell \mathbf{c} = \frac{\sigma(m+n) / \ell \mathbf{s}^\ell}{\ell(m+n) / \pi_0} + \frac{\sigma(m-n) / \ell \mathbf{s}^\ell}{\ell(m-n) / \pi_0} = 0$$

$$n > 0 \Rightarrow \int_{d\sigma/\ell}^{0|\ell} \sigma n / \ell \mathbf{c}^2 = \int_{d\sigma/\ell}^{0|\ell} \sigma n / \ell \mathbf{s}^2 = \frac{1}{2}$$

$$\int_{d\sigma/\ell}^{0|\ell} \sigma \gamma = \int_{d\sigma/\ell}^{0|\ell} \# \gamma_0 + \sqrt{2} \sum_{n>0} \sigma n / \ell \mathbf{c} \# \gamma_n = \int_{d\sigma/\ell}^{0|\ell} \# \gamma_0 = \# \gamma_0$$

$$m > 0 \Rightarrow \int_{d\sigma/\ell}^{0|\ell} \sqrt{2} \sigma m / \ell \mathbf{c} \sigma \gamma = \int_{d\sigma/\ell}^{0|\ell} \sqrt{2} \sigma m / \ell \mathbf{c} \left(\# \gamma_0 + \sqrt{2} \sum_{n>0} \sigma n / \ell \mathbf{c} \# \gamma_n \right) = 2 \int_{d\sigma/\ell}^{0|\ell} \sigma m / \ell \mathbf{c}^2 \# \gamma_m = \# \gamma_m$$

$$()^\mu = \int_{d\sigma/\ell}^{0|\ell} \sigma(\tau)^\mu - \tau_0^\sigma(\tau)^\mu : \quad {}_0()^\mu = \frac{\ell}{2\alpha'} \int_{d\sigma/\ell}^{0|\ell} \sigma_0(\tau)^\mu$$

$$()^\mu + \frac{2\alpha'}{\ell} {}_0()^\mu \tau = \int_{d\sigma/\ell}^{0|\ell} \sigma(\tau)^\mu : \quad \frac{2\alpha'}{\ell} {}_0()^\mu = \int_{d\sigma/\ell}^{0|\ell} \sigma_0(\tau)^\mu$$

$$()^\mu \times_0 ()^\nu = i^{\mu\nu} \eta$$

$$\begin{aligned} 2\alpha' ()^\mu \times_0 ()^\nu &= \int_{d\varrho/\ell}^{0|\ell} \underbrace{(\varrho(\tau)^\mu - \tau_0^\varrho(\tau)^\mu)}_{=} \times \int_{d\sigma/\ell}^{0|\ell} \varrho_0^\sigma(\tau)^\nu \\ &= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \underbrace{\varrho^\varrho(\tau)^\mu \times_0 \sigma^\sigma(\tau)^\nu}_{=2\ell\alpha' i^{\mu\nu} \frac{e\sigma}{\delta}} - \underbrace{\varrho \tau_0^\varrho(\tau)^\mu \times_0 \sigma^\sigma(\tau)^\nu}_{=0} = 2\alpha' i^{\mu\nu} \int_{d\sigma/\ell}^{0|\ell} 1 = 2\alpha' i^{\mu\nu} \eta \end{aligned}$$

$$\sqrt{2\alpha'} ()_n^\mu = e_{\mathbb{R}}^{\tau n} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \underbrace{-i\sqrt{n}^\sigma(\tau)^\mu + \varrho_0^\sigma(\tau)^\mu/\sqrt{n}}$$

$$\sqrt{2\alpha'} ()_n^{\ast\mu} = e_{\mathbb{R}}^{-\tau n} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \underbrace{i\sqrt{n}^\sigma(\tau)^\mu + \varrho_0^\sigma(\tau)^\mu/\sqrt{n}}$$

$$()_n^\mu e_{\mathbb{R}}^{-\tau n} - ()_n^{\ast\mu} e_{\mathbb{R}}^{\tau n} = \frac{-i}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \sigma(\tau)^\mu \sqrt{n}$$

$$()_n^\mu e_{\mathbb{R}}^{-\tau n} + ()_n^{\ast\mu} e_{\mathbb{R}}^{\tau n} = \frac{\varrho}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \varrho_0^\sigma(\tau)^\mu / \sqrt{n}$$

$$2()_n^\mu e_{\mathbb{R}}^{-\tau n} = \frac{-i}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \sigma(\tau)^\mu \sqrt{n} + \frac{\varrho}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \varrho_0^\sigma(\tau)^\mu / \sqrt{n}$$

$$2()_n^{\ast\mu} e_{\mathbb{R}}^{\tau n} = \frac{i}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \sigma(\tau)^\mu \sqrt{n} + \frac{\varrho}{\sqrt{\alpha'/2}} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \varrho_0^\sigma(\tau)^\mu / \sqrt{n}$$

$$()^\mu \times ()_n^\nu = 0: \quad {}_0()^\mu \times ()_n^\nu = 0$$

$$\begin{aligned}
()^\mu \times ()_n^\nu &= \int_{d\varrho/\ell}^{0|\ell} \underbrace{\varrho(\tau)^\mu - \tau_0^\varrho(\tau)^\mu}_{\varrho(\tau)^\mu - \tau_0^\varrho(\tau)^\mu} \times \int_{d\sigma/\ell}^{0|\ell} \underbrace{\sigma n/\ell \mathbf{c} \left(-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n} \right)}_{-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n}} \\
&= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \underbrace{\sigma n/\ell \mathbf{c} \left(\varrho(\tau)^\mu - \tau_0^\varrho(\tau)^\mu \right)}_{\varrho(\tau)^\mu - \tau_0^\varrho(\tau)^\mu} \times \underbrace{\left(-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n} \right)}_{-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n}} \\
&= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \underbrace{\left(-i^\varrho(\tau)^\mu \times {}^\sigma(\tau)^\nu \sqrt{n} + \ell^\varrho(\tau)^\mu \times {}^\sigma_0(\tau)^\nu / \sqrt{n} + i\tau_0^\varrho(\tau)^\mu \times {}^\sigma(\tau)^\nu \sqrt{n} - \ell\tau_0^\varrho(\tau)^\mu \times {}^\sigma_0(\tau)^\nu / \sqrt{n} \right)}_{\substack{=0 \\ = 2\ell i \alpha' \eta^{\mu\nu} \varrho^\sigma \\ = -2\pi i \alpha' \eta^{\mu\nu} \varrho^\sigma \\ =0}} \\
&= {}^{\mu\nu} \eta \alpha' \int_{d\sigma/\ell}^{0|\ell} \sigma n/\ell \mathbf{c} \underbrace{\left(i/\sqrt{n} - \tau \frac{2\pi}{\ell} \sqrt{n} \right)}_{i/\sqrt{n} - \tau \frac{2\pi}{\ell} \sqrt{n}} = 0
\end{aligned}$$

$$()^\mu_m \times ()_n^\nu = 0$$

$$\begin{aligned}
2\alpha' e^{-\tau(m+n)} ()^\mu_m \times ()_n^\nu &= \int_{d\varrho/\ell}^{0|\ell} \underbrace{\varrho m/\ell \mathbf{c} \left(-i^\varrho(\tau)^\mu \sqrt{m} + \ell_0^\varrho(\tau)^\mu / \sqrt{m} \right)}_{\varrho m/\ell \mathbf{c} \left(-i^\varrho(\tau)^\mu \sqrt{m} + \ell_0^\varrho(\tau)^\mu / \sqrt{m} \right)} \times \int_{d\sigma/\ell}^{0|\ell} \underbrace{\sigma n/\ell \mathbf{c} \left(-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n} \right)}_{-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n}} \\
&= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \varrho m/\ell \mathbf{c} \underbrace{\sigma n/\ell \mathbf{c} \left(-i^\varrho(\tau)^\mu \sqrt{m} + \ell_0^\varrho(\tau)^\mu / \sqrt{m} \right)}_{\varrho m/\ell \mathbf{c} \left(-i^\varrho(\tau)^\mu \sqrt{m} + \ell_0^\varrho(\tau)^\mu / \sqrt{m} \right)} \times \underbrace{\left(-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n} \right)}_{-i^\sigma(\tau)^\nu \sqrt{n} + \ell_0^\sigma(\tau)^\nu / \sqrt{n}} \\
&= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \varrho m/\ell \mathbf{c} \underbrace{\sigma n/\ell \mathbf{c} \left(-i^\varrho(\tau)^\mu \times {}^\sigma(\tau)^\nu \sqrt{mn} - \ell i_0^\varrho(\tau)^\mu \times {}^\sigma(\tau)^\nu \sqrt{\frac{n}{m}} - \ell i_0^\varrho(\tau)^\mu \times {}^\sigma_0(\tau)^\nu \sqrt{\frac{m}{n}} + \ell^{2\varrho}(\tau)^\mu \times {}^\sigma_0(\tau)^\nu / \sqrt{mn} \right)}_{\substack{=0 \\ = 2\ell \alpha' \eta^{\mu\nu} \varrho^\sigma \\ -2\ell \alpha' \eta^{\mu\nu} \varrho^\sigma \\ =0}} \\
&= 2\alpha' {}^{\mu\nu} \eta \underbrace{\left(\sqrt{\frac{m}{n}} - \sqrt{\frac{n}{m}} \int_{d\sigma/\ell}^{0|\ell} \sigma m/\ell \mathbf{c} \sigma n/\ell \mathbf{c} \right)}_{\delta_{mn}/2} = 0
\end{aligned}$$

$$()^{\mu}_m \times ()^{\nu}_n = {}^{\mu\nu}\eta \delta_{mn}$$

$$\begin{aligned}
2\alpha' e^{\tau(n-m)} ()^{\mu}_m \times ()^{\nu}_n &= \int_{d\varrho/\ell}^{0|\ell} \varrho^{m/\ell} \mathbf{c} \underbrace{-i^{\varrho}(\tau)^{\mu} \sqrt{m} + \ell_0^{\varrho}(\tau)^{\mu} / \sqrt{m}} \times \int_{d\sigma/\ell}^{0|\ell} \sigma^{n/\ell} \mathbf{c} \underbrace{i^{\sigma}(\tau)^{\nu} \sqrt{n} + \ell_0^{\sigma}(\tau)^{\nu} / \sqrt{n}} \\
&= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \varrho^{m/\ell} \mathbf{c} \sigma^{n/\ell} \mathbf{c} \underbrace{-i^{\varrho}(\tau)^{\mu} \sqrt{m} + \ell_0^{\varrho}(\tau)^{\mu} / \sqrt{m}} \times \underbrace{i^{\sigma}(\tau)^{\nu} \sqrt{n} + \ell_0^{\sigma}(\tau)^{\nu} / \sqrt{n}} \\
&= \int_{d\varrho/\ell}^{0|\ell} \int_{d\sigma/\ell}^{0|\ell} \varrho^{m/\ell} \mathbf{c} \sigma^{n/\ell} \mathbf{c} \underbrace{\left(\underbrace{i^{\varrho}(\tau)^{\mu} \times i^{\sigma}(\tau)^{\nu} \sqrt{mn}}_{=0} + \underbrace{\ell_0^{\varrho}(\tau)^{\mu} \times i^{\sigma}(\tau)^{\nu} \sqrt{\frac{n}{m}}}_{=2\ell\alpha'{}^{\mu\nu}\eta^{\varrho\sigma}} - \underbrace{\ell_0^{\varrho}(\tau)^{\mu} \times \ell_0^{\sigma}(\tau)^{\nu} \sqrt{\frac{m}{n}}}_{=-2\ell\alpha'{}^{\nu\mu}\eta^{\sigma\varrho}} + \underbrace{\ell_0^{2\varrho}(\tau)^{\mu} \times \ell_0^{\sigma}(\tau)^{\nu} / \sqrt{mn}}_{=0} \right)} \\
&= 2\alpha' {}^{\mu\nu}\eta \underbrace{\left(\sqrt{\frac{n}{m}} + \sqrt{\frac{m}{n}} \int_{d\sigma/\ell}^{0|\ell} \sigma^{m/\ell} \mathbf{c} \sigma^{n/\ell} \mathbf{c} \right)}_{\delta_{mn}/2} = 2\alpha' {}^{\mu\nu}\eta \delta_{mn}
\end{aligned}$$