

$$\left(P + \sum_i k_i \frac{y_i}{y}\right)^2 y - \sum_\ell (P + k_\ell + \dots + k_m)^2 (y_\ell - y_{\ell-1}) = \sum_{i < j} k_i \cdot k_j (y_j - y_i) \left(1 - \frac{y_j - y_i}{y}\right)$$

$$\begin{aligned} \text{LHS} &= P^2 y + \sum_i k_i^2 \frac{y_i^2}{y} + 2 \sum_i P \cdot k_i y_i + 2 \sum_{i < j} k_i \cdot k_j \frac{y_i y_j}{y} \\ &- P^2 \sum_\ell (y_\ell - y_{\ell-1}) - \sum_i k_i^2 \sum_{\ell \leq i} (y_\ell - y_{\ell-1}) - 2 \sum_i P \cdot k_i \sum_{\ell \leq i} (y_\ell - y_{\ell-1}) - 2 \sum_{i < j} k_i \cdot k_j \sum_{\ell \leq i} (y_\ell - y_{\ell-1}) \\ &= \sum_i k_i^2 \left(\frac{y_i^2}{y} - y_i\right) + 2 \sum_{i < j} k_i \cdot k_j \left(\frac{y_i y_j}{y} - y_i\right) = - \sum_{i < j} k_i \cdot k_j \left(\frac{y_i^2}{y} - y_i + \frac{y_j^2}{y} - y_j\right) \\ &+ 2 \sum_{i < j} k_i \cdot k_j \left(\frac{y_i y_j}{y} - y_i\right) = \sum_{i < j} k_i \cdot k_j \left(2 \frac{y_i y_j}{y} - 2y_i - \frac{y_i^2}{y} + y_i - \frac{y_j^2}{y} + y_j\right) = \text{RHS} \end{aligned}$$

$$\begin{aligned} &y \left(P + \sum_i k_i \frac{x_1 + \dots + x_i}{y}\right)^2 - \sum_\ell (P - k_1 + \dots + k_{\ell-1})^2 x_\ell \\ &= y \left(P + \sum_i k_i \frac{x_1 + \dots + x_i}{y}\right)^2 - \sum_\ell (P + k_\ell + \dots + k_m)^2 x_\ell \\ &= \sum_{i < j} k_i \cdot k_j (x_{i+1} + \dots + x_j) \left(1 - \frac{x_{i+1} + \dots + x_j}{y}\right) \end{aligned}$$

$$\int_{dP}^{1:d^{\mathbb{R}}} \prod_i \sqrt{x_i}^{(P - k_1 - \dots - k_{i-1})^2} = \prod_{i < j} \left(\frac{-1/2}{x_{i+1} \dots x_j} \exp \left(\frac{\log^2 x_{i+1} \dots x_j}{2y} \right) \right)^{k_i \cdot k_j}$$