$$m > 0: \quad 2\,L_m = \sum_{n}^{0|m} \alpha^{\mu}_{m-n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} + \sum_{n>m} \left(\alpha^{\mu}_{n} \underset{\mu\nu}{\eta} \overset{*}{\alpha}{}^{\nu}_{n-m} + \overset{*}{\alpha}{}^{\mu}_{n-m} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} \right)$$

$$2\,L_m = \sum_{n}^{\mathbb{Z}} \alpha^{\mu}_{m-n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} = \sum_{k<0} \alpha^{\mu}_{m-k} \underset{\mu\nu}{\eta} \alpha^{\nu}_{k} + \sum_{n}^{0|m} \alpha^{\mu}_{m-n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} + \sum_{n>m} \alpha^{\mu}_{m-n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n}$$

$$= \sum_{n}^{0|m} \alpha^{\mu}_{m-n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} + \sum_{n>m} \left(\alpha^{\mu}_{n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{\underbrace{m-n}_{=k}} + \alpha^{\mu}_{m-n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} \right) = \text{ RHS}$$

$$L_0 = \frac{1}{2} \alpha^{\mu}_{0} \underset{\mu\nu}{\eta} \alpha^{\nu}_{0} + \sum_{n>0} \overset{*}{\alpha}{}^{\mu}_{n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} = \frac{1}{2} \alpha_0 \cdot \alpha_0 + \sum_{n>0} \overset{*}{\alpha}_{n} \cdot \alpha_n$$

$$2\,L_0 = \alpha^{\mu}_{0} \underset{\mu\nu}{\eta} \alpha^{\nu}_{0} + 2 \sum_{n>0} \alpha^{\mu}_{-n} \underset{\mu\nu}{\eta} \alpha^{\nu}_{n} = \text{ RHS}$$

$$L_0 = \frac{1}{2} \underbrace{P^{\mu} \eta_{\mu\nu} P^{\nu}}\, ⧖\, {}^{>}\imath^{0} + \sum_{n>0}^{n} {}^{<}\imath^{n}\, ⧖\, \underbrace{\overset{*}{\partial}{}^{\mu} \eta_{\mu\nu} \partial^{\nu}}\, ⧖\, {}^{>}\imath^{n}$$

$$x^{L_0} = \sqrt{x}^{P^{\mu}\eta_{\mu\nu}P^{\nu}}\, ⧖ \bigotimes_{n}^{>0} x^{n \overset{*}{\partial}{}^{\mu} \eta_{\mu\nu} \partial^{\nu}}$$

$$L_m \times L_n = (m-n)\, L_{m+n} + \frac{d}{12} \left(m^3 - m \right) \delta_{m+n}$$

$${}^{2}\mathbb{R}_{2}^{\complement} \ni \begin{array}{c|c} a & b \\ \hline c & d \end{array} \text{ unbroken symm}$$