

$${}^\varphi x^j = {}^x \varphi^j \in \mathbb{C}$$

$${}^\varphi \bar{y}_i = {}^y \bar{\varphi}^i \in \mathbb{C}$$

$$\widehat{{}^\varphi dx^j} \varphi = {}^x \varphi^j$$

$$\widehat{{}^\varphi dy_i} \varphi = {}^y \bar{\varphi}^i$$

$$\widehat{{}^\varphi \partial_{sk} x^j} = {}^{s-x} \delta_k \delta^j$$

$$\widehat{{}^\varphi \partial_{sk} \bar{y}_i} = {}^{s-y} \delta^k \delta_i$$

$$\int \widehat{{}^\varphi \partial_{sk} x^j} {}^s \varphi^k = \widehat{{}^\varphi dx^j} \varphi = {}^x \varphi^j = \int {}^{s-x} \delta_k \delta^j {}^s \varphi^k$$

$$\int \widehat{{}^\varphi \partial_{sk} \bar{y}_i} {}^s \varphi^k = \widehat{{}^\varphi dy_i} \varphi = {}^y \bar{\varphi}^i = \int {}^{s-y} \delta^k \delta_i {}^s \varphi^k$$

$$\frac{d}{ds} \operatorname{sgn} (s-y) = 2\delta(s-y)$$

$$\int {}^s \varphi \partial \operatorname{sgn} (s-y) = - \int {}^s \widehat{\partial \varphi} \operatorname{sgn} (s-y) = \int_{-\infty}^y {}^s \widehat{\partial \varphi} - \int_x^\infty {}^s \widehat{\partial \varphi} = 2 {}^x \varphi = 2 \int {}^s \varphi \delta \operatorname{sgn} (s-y)$$

$$\frac{d}{ds}^{-1} \delta(s-y) = \frac{1}{2} \operatorname{sgn} (s-y)$$

$$\varphi \widehat{x^i \star_j \bar{y}} = \omega_\varphi \left(\widehat{{}^\varphi dx^i} \widehat{{}^\varphi d_j \bar{y}} \right) = \int \widehat{{}^\varphi \partial_{sk} x^i} \frac{d}{ds}^{-1} \widehat{{}^\varphi \partial_{sk} \bar{y}} = \int {}^{s-x} \delta_k \delta^i \frac{d}{ds}^{-1} {}^{s-y} \delta_i \delta^k$$

$$= {}_j \delta^i \int {}^{s-x} \delta \frac{d}{ds}^{-1} {}^{s-y} \delta = {}_j \delta^i \int {}^{s-x} \delta \frac{1}{2} \operatorname{sgn} (s-y) = {}_i \delta^j \frac{1}{2} \operatorname{sgn} (x-y)$$