$$m_1 \geqslant \cdot\cdot \geqslant m_\ell \geqslant 0$$

at most ℓ parts

$$P_0 = 1$$

$$P_n^{\leqslant \ell} = \sharp \frac{m_1 \geqslant \cdot\cdot \geqslant m_\ell \geqslant 0}{m_1 + \cdot\cdot + m_\ell = n}$$

$$\sum_{n \geqslant 0} x^n \, P_n^{\leqslant \ell} = \frac{1}{(1-x)\left(1-x^2\right) \cdot\cdot \left(1-x^\ell\right)}$$

$$\text{RHS} = (1-x)^{-1} \left(1-x^2\right)^{-1} \cdot\cdot \left(1-x^\ell\right)^{-1} = \sum_{k_1 \geqslant 0} x^{k_1} \sum_{k_2 \geqslant 0} \left(x^2\right)^{k_2} \cdots \sum_{k_\ell \geqslant 0} \left(x^\ell\right)^{k_\ell}$$

$$= \sum_{i}^{1|\ell} \sum_{k_i \geqslant 0} x^{k_1 + 2k_2 + \cdot\cdot + \ell k_\ell} = \sum_{n \geqslant 0} x^n \sum_{k_1 + 2k_2 + \cdot\cdot + \ell k_\ell = n} 1 = \text{ LHS}$$

$$\sum_{n \geqslant 0} x^n \, P_n = (1-x)^{-1} \left(1-x^2\right)^{-1} \cdot\cdot \left(1-x^\ell\right)^{-1} \cdots = \prod_{i \geqslant 0} \left(1-x^i\right)^{-1}$$

$$\sum_{n}^{\mathbb{N}} x^n \, P_n = \prod_{n \geqslant i} \frac{1}{1-x^i}$$

$$P_n \sim \frac{\exp\left(\pi\sqrt{2n/3}\right)}{4n\sqrt{3}}$$

$$P_n^{1/n} \sim \frac{\exp\left(\pi\sqrt{2/3n}\right)}{\left(4n\sqrt{3}\right)^{1/n}} \sim 1$$