$$\text{free } S \subset \bar{\mathsf{L}} \Rightarrow \bigvee_{\text{basis}} S \subset B \subset \bar{\mathsf{L}}$$

$$S \in \mathcal{S} = \frac{T \subset \bar{\mathsf{L}}}{S \subset T \text{ free}} \neq \varnothing$$

$$\mathcal{S} \text{ inclusion ord } T \subset \acute{T}$$

$$\text{tot ord } \varnothing \neq \mathcal{K} \subset \mathcal{S} \underset{\text{ob Schr}}{\Longrightarrow} \overset{\mathcal{K}}{\cup} = \bigcup_{T}^{\mathcal{K}} T \in \mathcal{S}$$

$$\overset{\mathcal{K}}{\cup} \text{ free : } \begin{cases} {}_1\mathsf{L} \cdots {}_n\mathsf{L} \in \overset{\mathcal{K}}{\cup} \\ \alpha^i \, {}_i\mathsf{L} = 0 \end{cases} \Rightarrow {}_i\mathsf{L} \in T_i \in \mathcal{K} \underset{\text{tot ord}}{\Longrightarrow} \bigvee_j {}_1\mathsf{L} \cdots {}_n\mathsf{L} \in T_j \text{ free} \Rightarrow \alpha^1 = \cdots = \alpha^n = 0$$

$$S \subset \overset{\mathcal{K}}{\cup} : \quad \bigvee T \in \mathcal{K} \Rightarrow S \subset T \subset \overset{\mathcal{K}}{\cup}$$

$$\underset{\text{Zorn}}{\Longrightarrow} \bigvee B \underset{\max}{\in} \mathcal{S} \Rightarrow S \subset B \text{ free } \lightning \, B \text{ not full } \Rightarrow \bigvee \mathbb{L} \in \bar{\mathsf{L}} \setminus < B >$$

$$\mathbb{L} \cup B \text{ free}$$

$$\sum_b^{B \text{ fin}} \lambda_b b + \lambda \mathsf{L} = 0$$

$$\lightning \, \lambda \neq 0 \Rightarrow \mathsf{L} = -\sum_b^{B \text{ fin}} \frac{\lambda_b}{\lambda} b \in < B > \lightning \Rightarrow \lambda = 0 \Rightarrow \sum_b^{B \text{ fin}} \lambda_b b = 0 \Rightarrow \bigwedge_b^{B} \lambda_b = 0$$

$$B \underset{\neq}{\subset} \mathbb{L} \cup B \in \mathcal{S} \lightning \Rightarrow B \text{ full } \Rightarrow B \text{ basis}$$

$$\begin{cases} S \text{ free} \\ T \text{ full} \end{cases} \Rightarrow \bigvee S \xrightarrow[\text{inj}]{\psi} T$$

$$\varnothing \in \mathcal{M} = \frac{A{:}\varphi}{S \supset A \xrightarrow[\text{inj}]{\varphi} T \llcorner \underbrace{S \llcorner A}: \quad {}^{A}\varphi \cup \underbrace{S \llcorner A} \text{ free}} \neq \varnothing$$

$$\text{order } \underbrace{A{:}\varphi} \prec \underbrace{B{:}\psi} \Leftrightarrow \begin{array}{ccc} A & & \\ & \searrow^{\varphi} & \\ \cap & & T \\ & \nearrow_{\psi} & \\ B & & \end{array}$$

$$\text{tot ord } \varnothing \neq \mathcal{K} \subset \mathcal{M} \underset{\text{ob Schr}}{\Longrightarrow} \underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A : \bigcup_{A:\varphi}^{\mathcal{K}} \varphi} \in \mathcal{M}$$

$$\bigcup_{A:\varphi}^{\mathcal{K}} A \subset S$$

$$\bigwedge_{B:\psi}^{\mathcal{K}} {}^{B}\overleftarrow{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} = \psi \Rightarrow \bigcup_{A:\varphi}^{\mathcal{K}} A \xrightarrow[\text{well-def}]{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} T$$

$${}^{\underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A}}\overbrace{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} = \bigcup_{A:\varphi}^{\mathcal{K}} {}^{A}\varphi$$

$$\bigwedge_{B:\psi}^{\mathcal{K}} {}^{B}\overbrace{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} = {}^{B}\psi \subset T\llcorner \underbrace{S\llcorner B} \subset T\llcorner \overbrace{S\llcorner \underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A}} \Rightarrow {}^{\underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A}}\overbrace{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} \subset T\llcorner \overbrace{S\llcorner \underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A}}$$

$${}^{\underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A}}\overbrace{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} \cup \overbrace{S\llcorner \underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A}} \text{ free}$$

$$\frac{{}_{1}\lfloor \cdots {}_{m}\lfloor \in \bigcup_{A:\varphi}^{\mathcal{K}} A}{{}_{1}\urcorner \cdots {}_{n}\urcorner \in S\llcorner \underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A}} \sum_{i} \alpha^{i} \; {}^{{}_{i}\lfloor}\overbrace{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} + \sum_{j} \beta^{j} \; {}_{j}\urcorner = 0$$

$$\Rightarrow \bigvee_{B:\psi}^{\mathcal{K}} {}_{1}\lfloor \cdots {}_{m}\lfloor \in B \Rightarrow {}^{{}_{i}\lfloor}\overbrace{\bigcup_{A:\varphi}^{\mathcal{K}} \varphi} = {}^{{}_{i}\lfloor}\psi \in {}^{B}\psi$$

$${}_{1}\urcorner \cdots {}_{n}\urcorner \in S\llcorner \underbrace{\bigcup_{A:\varphi}^{\mathcal{K}} A} \subset S\llcorner B \underset{{}^{B}\psi \cup \underbrace{S\llcorner B} \text{ free}}{\Longrightarrow} \bigwedge_{i:j} \alpha^{i} = 0 = \beta^{j}$$

$$\text{Zorn } \bigvee B:\psi \underset{\max}{\in} \mathcal{M} \Rightarrow B \subset S$$

$$\lightning\ B \neq S \Longrightarrow \bigvee a \in S \setminus B \underset{{}^{B}\psi \,\cup\, \underbrace{S \setminus B} \text{ free}}{\Longrightarrow} a \notin < {}^{B}\psi > \Longrightarrow {}^{B}\psi \text{ not full } \Longrightarrow T \not\subset < {}^{B}\psi > \Longrightarrow \bigvee b \in T \setminus < {}^{B}\psi >$$

$$b \in < \underbrace{S \setminus B} \cup {}^{B}\psi >$$

$$\lightning\ b \notin < \underbrace{S \setminus B} \cup {}^{B}\psi > \Longrightarrow \underbrace{S \setminus B} \cup {}^{B}\psi \cup b \text{ free } \Longrightarrow B{:}b \underset{\neq}{\prec} \underbrace{B \cup b}{:} \begin{bmatrix} \psi \\ a \mapsto b \end{bmatrix} \in \mathcal{M}\ \lightning$$

$$< \underbrace{S \setminus B} \cup {}^{B}\psi > \ni b \underset{\text{eind}}{=} \sum_{s}^{S \setminus B} \lambda_s s + \sum_{t}^{B} \mu_t\, {}^{t}\psi$$

$$b \notin < {}^{B}\psi > \Longrightarrow \bigvee_{c}^{S \setminus B} \lambda_c \neq 0 \underset{\text{eind}}{\Longrightarrow} b \notin < \overbrace{S \setminus \underbrace{B \cup c}} \cup {}^{B}\psi > \Longrightarrow \overbrace{S \setminus \underbrace{B \cup c}} \cup {}^{B}\psi \cup b \text{ free } \Longrightarrow \underbrace{B{:}\psi} \underset{\neq}{\prec} \underbrace{B \cup c}{:} \begin{bmatrix} \psi \\ c \mapsto b \end{bmatrix}\ \lightning$$

$$B = S \Longrightarrow S \xrightarrow[\text{inj}]{\psi} T$$