$$a \wedge b = ab$$

$$a \vee b = a + b - ab$$

$$\bar{a} = e - a$$

$$\overline{a \vee b} = e - a \vee b = \underline{e-a}\,\underline{e-b} = \bar{a} \wedge \bar{b}$$

$$\begin{cases} a \wedge b = b \wedge a \\ a \vee b = b \vee a \end{cases}$$

$$a \wedge b = ab \underset{\text{comm}}{=} ba = b \wedge a$$

$$e - a \vee b = \underline{e-a}\,\underline{e-b} \underset{\text{comm}}{=} \underline{e-b}\,\underline{e-a} = e - b \vee a$$

$$\begin{cases} \underline{a \wedge b} \wedge c = a \wedge \underline{b \wedge c} \\ \underline{a \vee b} \vee c = a \vee \underline{b \vee c} \end{cases}$$

$$\underline{a \wedge b} \wedge c = \underline{ab}\,c \underset{\text{ass}}{=} a\,\underline{bc} = a \wedge \underline{b \wedge c}$$

$$e - \overline{a \vee b \vee c} = \underline{e - a \vee b}\,\underline{e-c} = \overline{\underline{e-a}\,\underline{e-b}}\,\underline{e-c} \underset{\text{ass}}{=} \underline{e-a}\,\overline{\underline{e-b}\,\underline{e-c}} = \underline{e-a}\,\underline{e-b \vee c} = e - \overline{a \vee \underline{b \vee c}}$$

$$a \wedge a = a = a \vee a$$

$$a \wedge a = a^2 = a$$

$$a \vee a = a + a - a^2 = a + a - a = a$$

$$\underline{a \vee b} \wedge a = a = \underline{a \wedge b} \vee a$$

$$\underline{a \vee b} \wedge a = \underline{a + b - ab}\,a = a^2 + ba - ba^2 = a + ba - ba = a$$

$$\underline{a \wedge b} \vee a = ab + a - \underline{ab}\,a = ab + a - a^2 b = ab + a - ab = a$$

$$o \leqslant \mathbb{1} \leqslant e$$

$$\mathbb{1} \wedge e = \mathbb{1}e = \mathbb{1} \implies \mathbb{1} \leqslant e$$

$$\mathbb{1} \wedge o = \mathbb{1}o = o \implies o \leqslant \mathbb{1}$$

$$\underbrace{\mathbb{1} \wedge \#} \vee \underbrace{\# \wedge \#} = \underbrace{\mathbb{1} \vee \#} \wedge \#$$

$$\underbrace{\mathbb{1} \wedge \#} \vee \underbrace{\# \wedge} = \mathbb{1}\# \vee \#\# = \mathbb{1}\# + \#\# - \mathbb{1}\#\#\# = \mathbb{1}\# + \#\# - \mathbb{1}\#\#^2 = \mathbb{1} + \#\# - \mathbb{1}\#\# = \underbrace{\mathbb{1} + \# - \mathbb{1}\#}\# = \underbrace{\mathbb{1} \vee \#} \wedge \#$$

$$\begin{cases} \mathbb{1} \vee \bar{\mathbb{1}} = e \\ \mathbb{1} \wedge \bar{\mathbb{1}} = o \end{cases}$$

$$\mathbb{1} \vee \bar{\mathbb{1}} = \mathbb{1} \vee \underline{e\text{-}\mathbb{1}} = \mathbb{1} + \underline{e\text{-}\mathbb{1}} - \mathbb{1}\underline{e\text{-}\mathbb{1}} = \mathbb{1} + e\text{-}\mathbb{1} - \mathbb{1} + \mathbb{1}^2 = \mathbb{1} + e\text{-}\mathbb{1} - \mathbb{1} + \mathbb{1} = e$$

$$\mathbb{1} \wedge \bar{\mathbb{1}} = \mathbb{1} \wedge \underline{e\text{-}\mathbb{1}} = \mathbb{1}\underline{e\text{-}\mathbb{1}} = \mathbb{1} - \mathbb{1}^2 = \mathbb{1} - \mathbb{1} = o$$