

\mathbb{Z} Boolat

$$2 \triangleleft_{\mathbb{Z}} = \frac{2 \xleftarrow{k} \mathbb{Z}}{\text{hom}}$$

$$\underbrace{k \mathbb{Z} \wedge \mathbb{Z}} = \underbrace{k \mathbb{Z}} \underbrace{k \mathbb{Z}}; \quad k \bar{\mathbb{Z}} = 1 - k \mathbb{Z}$$

$$\underbrace{k \mathbb{Z} \vee \mathbb{Z}} = \overline{k \bar{\mathbb{Z}} \wedge \bar{\mathbb{Z}}} = 1 - \underbrace{k \bar{\mathbb{Z}} \wedge \bar{\mathbb{Z}}} = 1 - \underbrace{k \bar{\mathbb{Z}}} \underbrace{\bar{\mathbb{Z}}} = 1 - \underbrace{1 - k \mathbb{Z}} \underbrace{1 - k \mathbb{Z}} = k \mathbb{Z} \underset{\text{max}}{\vee} k \mathbb{Z}$$

$$2 \triangleleft_{\mathbb{Z}} \in \triangleleft_0^0 \text{ tot unzush}$$

$$\mathbb{Z} \sqsubset_{\text{ideal}} \mathbb{Z}$$

$$\max \mathbb{Z} \subset \mathbb{Z} \sqsubset_{\text{ideal}} \mathbb{Z} \Rightarrow \mathbb{Z} = \mathbb{Z}$$

$$2 \triangleleft_{\mathbb{Z}} \ni k \Rightarrow \ker k \underset{\text{ideal}}{\sqsubset} \mathbb{Z}$$

$$\ker k \sqsubset_{\text{ideal}} \mathbb{Z}$$

$$k e = 1 \neq 0 \Rightarrow e \notin \ker k$$

$$\ker k \subset \mathbb{Z} \sqsubset_{\text{ideal}} \mathbb{Z} \Rightarrow \bigvee_{\mathbb{Z}}^{\mathbb{Z} \setminus \ker k} \Rightarrow k \mathbb{Z} \neq 0 \Rightarrow k \mathbb{Z} = 1 = k e \Rightarrow \mathbb{Z} \underset{\ker k}{\sim} e \Rightarrow \mathbb{Z} \wedge \bar{e} \in \ker k \ni e \wedge \bar{\mathbb{Z}}$$

$$\Rightarrow \mathbb{Z} \wedge \bar{e} \in \mathbb{Z} \ni e \wedge \bar{\mathbb{Z}} \Rightarrow e \underset{\mathbb{Z}}{\sim} \mathbb{Z} \underset{\mathbb{Z}}{\sim} o \xrightarrow{\text{trans}} e \underset{\mathbb{Z}}{\sim} o \Rightarrow e \in \mathbb{Z} \Rightarrow \mathbb{Z} = \mathbb{Z} \Rightarrow \ker k \text{ max}$$

$$\max_{\mathfrak{I} \text{ ideal}} \mathfrak{I} \subseteq \mathfrak{J} \Rightarrow \bigwedge_{\mathfrak{I}} \vee \begin{cases} \mathfrak{I} \in \mathfrak{I} \\ \bar{\mathfrak{I}} \in \mathfrak{I} \end{cases}$$

$$\mathfrak{I} \not\subseteq \mathfrak{I} \text{ If } \mathfrak{I} = e \Rightarrow \bar{\mathfrak{I}} = o \in \mathfrak{I}$$

$$\text{If } \mathfrak{I} \neq e \Rightarrow \mathfrak{I} \subset \langle \mathfrak{I} \cup \mathfrak{I} \rangle = \frac{\mathfrak{I} \in \mathfrak{I}}{\bigvee_{\mathfrak{I}_1 \dots \mathfrak{I}_n} \mathfrak{I} \leq \mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_n} \stackrel{\text{ideal}}{\subseteq} \mathfrak{I} \stackrel{\text{max}}{\Rightarrow} \langle \mathfrak{I} \cup \mathfrak{I} \rangle = \mathfrak{I}$$

$$\Rightarrow e \in \langle \mathfrak{I} \cup \mathfrak{I} \rangle \Rightarrow \bigvee_{\mathfrak{I}_1 \dots \mathfrak{I}_n} e = \mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_n$$

$$\nexists \mathfrak{I}_1 \dots \mathfrak{I}_n \in \mathfrak{I} \Rightarrow e = \mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_n \in \mathfrak{I} \Rightarrow \mathfrak{I} = \mathfrak{I} \nexists$$

$$\nexists \mathfrak{I}_1 = \dots = \mathfrak{I}_n = \mathfrak{I} \Rightarrow \mathfrak{I} = \mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_n \in \mathfrak{I} \nexists$$

$$\stackrel{\text{OE}}{\Rightarrow} \begin{cases} 1 \leq m & \mathfrak{I}_1 \dots \mathfrak{I}_m \in \mathfrak{I} \\ m < n & \mathfrak{I}_{m+1} = \dots = \mathfrak{I}_n = \mathfrak{I} \end{cases} \Rightarrow e = \underbrace{\mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_m}_{\in \mathfrak{I}} \vee \mathfrak{I}$$

$$\Rightarrow \bar{\mathfrak{I}} = e \wedge \bar{\mathfrak{I}} = \overline{\mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_m \vee \mathfrak{I}} \wedge \bar{\mathfrak{I}} \stackrel{\text{distr}}{=} \overline{\mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_m} \wedge \bar{\mathfrak{I}} \vee \underbrace{\mathfrak{I} \wedge \bar{\mathfrak{I}}}_{=o} = \overline{\mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_m} \wedge \bar{\mathfrak{I}} \leq \mathfrak{I}_1 \vee \dots \vee \mathfrak{I}_m \in \mathfrak{I} \\ \Rightarrow_{\text{ideal}} \bar{\mathfrak{I}} \in \mathfrak{I}$$

$$\bigwedge_{\mathfrak{I}} \vee \begin{cases} \mathfrak{I} \in \mathfrak{I} \\ \bar{\mathfrak{I}} \in \mathfrak{I} \end{cases} \Rightarrow \bigwedge_{\mathfrak{I}: \mathfrak{I}} \mathfrak{I} \wedge \bar{\mathfrak{I}} \in \mathfrak{I} \Rightarrow \vee \begin{cases} \mathfrak{I} \in \mathfrak{I} \\ \bar{\mathfrak{I}} \in \mathfrak{I} \end{cases}$$

$$\mathfrak{I} \notin \mathfrak{I} \nexists \bar{\mathfrak{I}} \Rightarrow \bar{\mathfrak{I}} \in \mathfrak{I} \ni \bar{\mathfrak{I}} \Rightarrow \overline{\mathfrak{I} \wedge \bar{\mathfrak{I}}} = \bar{\mathfrak{I}} \vee \bar{\bar{\mathfrak{I}}} \in \mathfrak{I} \stackrel{e \notin \mathfrak{I}}{\Rightarrow} \mathfrak{I} \wedge \bar{\mathfrak{I}} \notin \mathfrak{I}$$

$$\bigwedge_{\gamma: \Psi} \gamma \wedge \Psi \in \mathfrak{I} \Rightarrow \vee \begin{cases} \gamma \in \mathfrak{I} \\ \Psi \in \mathfrak{I} \end{cases} \Rightarrow \bigvee_k \ker k = \mathfrak{I}$$

$$2 \xleftarrow{k} \mathfrak{I}$$

$$k\gamma = \begin{cases} 0 & \gamma \in \mathfrak{I} \\ 1 & \gamma \notin \mathfrak{I} \end{cases}$$

$$k(\gamma \wedge \Psi) = (k\gamma) \cdot (k\Psi)$$

$$\text{If } k(\gamma \wedge \Psi) = 0 \Rightarrow \gamma \wedge \Psi \in \mathfrak{I} \Rightarrow \vee \begin{cases} \gamma \in \mathfrak{I} \Rightarrow k\gamma = 0 \\ \Psi \in \mathfrak{I} \Rightarrow k\Psi = 0 \end{cases} \Rightarrow (k\gamma) \cdot (k\Psi) = 0$$

$$\text{If } k(\gamma \wedge \Psi) = 1 \Rightarrow \gamma \wedge \Psi \notin \mathfrak{I}$$

$$\gamma \geq \gamma \wedge \Psi \leq \Psi \xrightarrow{\text{ideal}} \gamma \notin \mathfrak{I} \not\leq \Psi \Rightarrow k\gamma = 1 = k\Psi \Rightarrow (k\gamma) \cdot (k\Psi) = 1$$

$$k\bar{\gamma} = 1 - k\gamma$$

$$\text{If } k\gamma = 0 \Rightarrow \gamma \in \mathfrak{I} \xrightarrow{e \notin \mathfrak{I}} \bar{\gamma} \notin \mathfrak{I} \Rightarrow k\bar{\gamma} = 0 = 1 - k\gamma$$

$$\text{If } k\gamma = 1 \Rightarrow \gamma \notin \mathfrak{I}$$

$$\gamma \wedge \bar{\gamma} = 0 \in \mathfrak{I} \xrightarrow{\text{Vor}} \bar{\gamma} \in \mathfrak{I} \Rightarrow k\bar{\gamma} = 0 = 1 - k\gamma$$

$$\Rightarrow 2 \xleftarrow[\text{hom}]{k} \mathfrak{I}$$

$$\ker k = \mathfrak{I}$$

$$e \notin \mathfrak{I} \subset_{\text{ideal}} \mathbb{Z} \ni \mathfrak{I} \text{ Boolat} \Rightarrow \bigvee_{\text{max}} \mathfrak{I} \subset \mathfrak{m} \stackrel{\text{max}}{\mathbb{F}}_{\text{ideal}} \mathfrak{I}$$

$$\mathfrak{I} \in \mathcal{J} = \frac{\mathfrak{I} \cap \mathfrak{I}}{\mathfrak{I} \cap \mathfrak{I}} \neq \emptyset$$

$$\text{order : } \mathfrak{I} \prec \mathfrak{I} \leftrightarrow \mathfrak{I} \subset \mathfrak{I}$$

$$\text{Kette=tot ord nonvoid } \mathcal{C} \subset \mathcal{J} \Rightarrow \bigwedge_{\mathfrak{I} : \mathfrak{I}}^{\mathcal{C}} \vee \left\{ \begin{array}{l} \mathfrak{I} \subset \mathfrak{I} \\ \mathfrak{I} \supset \mathfrak{I} \end{array} \right.$$

$$e \notin \bigcup \mathcal{C} = \frac{\mathfrak{I} \in \mathfrak{I}}{\bigvee_{\mathfrak{I}} \mathfrak{I} \in \mathfrak{I}} \subset_{\text{ideal}} \mathfrak{I}$$

$$\mathcal{C} \geq \bigcup \mathcal{C} \in \mathcal{J} \text{ ob Schr}$$

$$\mathfrak{I} \in \mathfrak{I} \ni \mathfrak{I} \Rightarrow \bigvee_{\mathfrak{I} : \mathfrak{I}}^{\mathcal{C}} \left\{ \begin{array}{l} \mathfrak{I} \in \mathfrak{I} \\ \mathfrak{I} \in \mathfrak{I} \end{array} \right. \xrightarrow{\text{OE}} \mathfrak{I} \subset \mathfrak{I} \Rightarrow \mathfrak{I} \vee \mathfrak{I} \in \mathfrak{I} \subset \bigcup \mathcal{C}$$

$$\bigcup \mathcal{C} \ni \mathfrak{I} \geq \mathfrak{I} \Rightarrow \bigvee_{\mathfrak{I}}^{\mathcal{C}} \mathfrak{I} \in \mathfrak{I} \Rightarrow \mathfrak{I} \in \mathfrak{I} \subset \bigcup \mathcal{C}$$

$$\mathcal{C} \neq \emptyset \Rightarrow \bigvee_{\mathfrak{I}}^{\mathcal{C}} \Rightarrow o \in \mathfrak{I} \subset \bigcup \mathcal{C}$$

$$\bigwedge_{\mathfrak{I}}^{\mathcal{C}} e \notin \mathfrak{I} \Rightarrow e \notin \bigcup \mathcal{C}$$

$$\xrightarrow{\text{Zorn}} \max \bigvee_{\mathfrak{m}}^{\mathcal{J}} \Rightarrow \mathfrak{I} \subset \mathfrak{m} \stackrel{\text{max}}{\mathbb{F}}_{\text{ideal}} \mathfrak{I} \Rightarrow \left\{ \begin{array}{l} \forall \mathfrak{k} \in \mathbb{Z} \ni \mathfrak{I} \\ \ker \mathfrak{k} = \mathfrak{m} \end{array} \right.$$

$$\mathbb{N} \ni \mathbb{1} \text{ BoolLat} \Rightarrow 2^{\mathbb{N}}_{\mathbb{1}} \neq \emptyset$$

$$\text{BoolLat } \mathbb{N} \ni \mathbb{1} \ni \gamma \neq e \Rightarrow \begin{cases} \forall k \in 2^{\mathbb{N}}_{\mathbb{1}} \\ k\gamma = 0 \end{cases}$$

$$e \notin \langle \gamma \rangle = \frac{\{\gamma \in \mathbb{1}\}}{\{\gamma \geq \gamma\}} \sqsubset_{\text{ideal}} \mathbb{1} \Rightarrow \bigvee \langle \gamma \rangle \sqsubset \mathfrak{m} \stackrel{\max}{\sqsubset}_{\text{ideal}} \mathbb{1} \Rightarrow \begin{cases} \forall k \in 2^{\mathbb{N}}_{\mathbb{1}} \\ \ker k = \mathfrak{m} \end{cases} \Rightarrow k\gamma = 0$$