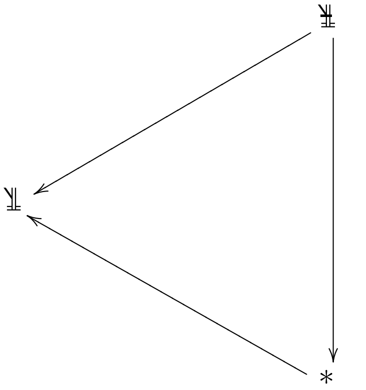
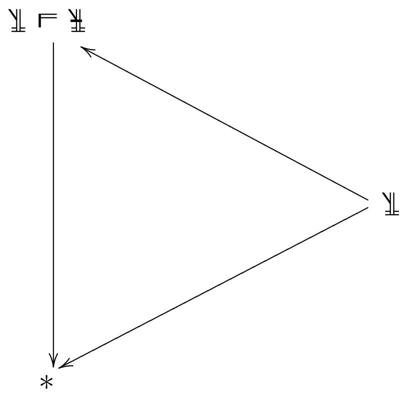
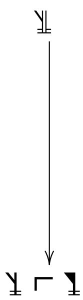




$\sqsubset_{\text{ideal}}$ Boolat

$\sqcap$



$$a \sim b \Leftrightarrow \bigvee_{x:y}^{I} a \vee x = b \vee y \text{ equ-rel}$$

$$\text{refl}: \quad a \vee \underset{\in I}{o} = a \vee o \Longrightarrow a \sim a$$

$$\text{symm}: \quad a \vee x = b \vee y \Longrightarrow b \vee y = a \vee x$$

$$\text{trans}: \quad a \sim b \sim c \Longrightarrow \begin{cases} \bigvee\limits_{x:y}^{I} & a \vee x = b \vee y \\ \bigvee\limits_{z:w'}^{I} & b \vee z = c \vee w' \end{cases} \Longrightarrow$$

$$a \vee \underbrace{x \vee z}_{\in I} = \underbrace{a \vee x} \vee z = \underbrace{b \vee y} \vee z = b \vee \underbrace{y \vee z} = b \vee \underbrace{z \vee y}$$

$$= \underbrace{b \vee z} \vee y = \underbrace{c \vee w'} \vee y = c \vee \underbrace{w' \vee y}_{\in I} \Longrightarrow a \sim c$$

$$\sim \text{ Kongr-Rel}$$

$$\text{neg}: \quad a \sim b \Longrightarrow \bigvee_{x:y}^{I} a \vee x = b \vee y \Longrightarrow \bar{a} \wedge \bar{x} = \overline{a \vee x} = \overline{b \vee y} = \bar{b} \wedge \bar{y} \Longrightarrow$$

$$\bar{a} \vee \underbrace{x \vee y} = \underbrace{\bar{a} \vee x \vee y} \wedge \underbrace{\bar{x} \vee x \vee y}_{=e} \underset{\text{distr}}{=} \underbrace{\bar{a} \wedge \bar{x}} \vee \underbrace{x \vee y} = \underbrace{\bar{b} \wedge \bar{y}} \vee \underbrace{x \vee y} = \underbrace{\bar{b} \vee x \vee y} \wedge \underbrace{\bar{y} \vee x \vee y} = \bar{b} \vee \underbrace{x \vee y}$$

$$\underset{x \vee y \in I}{\Longrightarrow} \bar{a} \sim \bar{b}$$

$$\vee: \quad \begin{cases} a \sim b & \Longrightarrow \bigvee\limits_{x:y}^{I} a \vee x = b \vee y \\ c \sim d & \Longrightarrow \bigvee\limits_{z:w'}^{I} c \vee z = d \vee w' \end{cases} \Longrightarrow \underbrace{a \vee c} \vee \underbrace{x \vee z}_{\in I} = \underbrace{a \vee x} \vee \underbrace{c \vee z} = \underbrace{b \vee y} \vee \underbrace{d \vee w'} = \underbrace{b \vee d} \vee \underbrace{y \vee w'}_{\in I}$$

$$\Longrightarrow a \vee c \sim b \vee d$$

$$\wedge: \quad \begin{cases} a \sim b & \Longrightarrow \bar{a} \sim \bar{b} \\ c \sim d & \Longrightarrow \bar{c} \sim \bar{d} \end{cases} \Longrightarrow \bar{a} \vee \bar{c} \sim \bar{b} \vee \bar{d} \Longrightarrow a \wedge c = \overline{\bar{a} \vee \bar{c}} \sim \overline{\bar{b} \vee \bar{d}} = b \wedge d$$

$$\mathfrak{I} = \frac{\mathfrak{a} \in \mathfrak{L}}{\mathfrak{a} \sim o}$$

$$\subset : \quad \mathfrak{a} \in \mathfrak{I} \Rightarrow \mathfrak{a} \vee \mathfrak{a} = \mathfrak{a} = \mathfrak{a} \vee o \Rightarrow \mathfrak{a} \sim o$$

$$\supset : \quad \mathfrak{a} \sim o \Rightarrow \bigvee_{\mathfrak{c} : \mathfrak{d}}^{\mathfrak{I}} \mathfrak{a} \vee \mathfrak{c} = o \vee \mathfrak{d} = \mathfrak{d} \Rightarrow \mathfrak{a} \leqslant \mathfrak{a} \vee \mathfrak{c} = \mathfrak{d} \in \mathfrak{I} \Rightarrow \mathfrak{a} \in \mathfrak{I}$$

$$\mathfrak{a} \sim \mathfrak{b} \Leftrightarrow \mathfrak{a} \wedge \bar{\mathfrak{b}} \in \mathfrak{I} \ni \mathfrak{b} \wedge \bar{\mathfrak{a}}$$

$$\Leftarrow : \quad \mathfrak{a} \vee \underbrace{\bar{\mathfrak{a}} \wedge \mathfrak{b}}_{\in \mathfrak{I}} \underset{\text{distr}}{=} \underbrace{\mathfrak{a} \vee \bar{\mathfrak{a}}}_{= e} \wedge \underbrace{\mathfrak{a} \vee \mathfrak{b}} = \mathfrak{a} \vee \mathfrak{b} = \mathfrak{b} \vee \mathfrak{a} = \underbrace{\mathfrak{b} \vee \bar{\mathfrak{b}}}_{= e} \wedge \underbrace{\mathfrak{b} \vee \mathfrak{a}} \underset{\text{distr}}{=} \mathfrak{b} \vee \underbrace{\bar{\mathfrak{b}} \wedge \mathfrak{a}}_{\in \mathfrak{I}} \Rightarrow \mathfrak{a} \sim \mathfrak{b}$$

$$\Rightarrow : \quad \mathfrak{a} \sim \mathfrak{b} \Rightarrow \bigvee_{\mathfrak{c} : \mathfrak{d}}^{\mathfrak{I}} \mathfrak{a} \vee \mathfrak{c} = \mathfrak{b} \vee \mathfrak{d} \Rightarrow \mathfrak{a} \wedge \bar{\mathfrak{b}} \leqslant \mathfrak{a} \leqslant \mathfrak{a} \vee \mathfrak{c} \Rightarrow \mathfrak{a} \wedge \bar{\mathfrak{b}} = \underbrace{\mathfrak{a} \wedge \bar{\mathfrak{b}}} \wedge \underbrace{\mathfrak{a} \vee \mathfrak{c}} = \underbrace{\mathfrak{a} \wedge \bar{\mathfrak{b}}} \wedge \underbrace{\mathfrak{b} \vee \mathfrak{d}}$$

$$\underset{\text{distr}}{=} \underbrace{\mathfrak{a} \wedge \bar{\mathfrak{b}} \wedge \mathfrak{b}}_{= o} \vee \underbrace{\mathfrak{a} \wedge \bar{\mathfrak{b}} \wedge \mathfrak{d}} = o \vee \underbrace{\mathfrak{a} \wedge \bar{\mathfrak{b}} \wedge \mathfrak{d}} = \mathfrak{a} \wedge \bar{\mathfrak{b}} \wedge \mathfrak{d} \leqslant \mathfrak{d} \in \mathfrak{I} \underset{\text{ideal}}{\Rightarrow} \mathfrak{a} \wedge \bar{\mathfrak{b}} \in \mathfrak{I} \ni \mathfrak{b} \wedge \bar{\mathfrak{a}} \text{ analog}$$

$$\sim \text{ Kongr-Rel } \Rightarrow \mathfrak{a} \sim \mathfrak{b} \Leftrightarrow \mathfrak{a} \wedge \bar{\mathfrak{b}} \in \tilde{\mathfrak{L}}^{o} \ni \mathfrak{b} \wedge \bar{\mathfrak{a}}$$

$$\Rightarrow : \quad \mathfrak{a} \sim \mathfrak{b} \underset{\wedge\,\text{congr}}{\Rightarrow} \mathfrak{a} \wedge \bar{\mathfrak{b}} \sim \mathfrak{b} \wedge \bar{\mathfrak{b}} = o \Rightarrow \mathfrak{a} \wedge \bar{\mathfrak{b}} \in \tilde{\mathfrak{L}}^{o} \ni \mathfrak{b} \wedge \bar{\mathfrak{a}} \text{ analog}$$

$$\Leftarrow : \quad \mathfrak{a} \wedge \bar{\mathfrak{b}} \in \tilde{\mathfrak{L}}^{o} \ni \mathfrak{b} \wedge \bar{\mathfrak{a}} \Rightarrow \mathfrak{a} \wedge \bar{\mathfrak{b}} \sim o \sim \mathfrak{b} \wedge \bar{\mathfrak{a}} \underset{\text{trans}}{\Rightarrow} \mathfrak{a} \wedge \bar{\mathfrak{b}} \sim \mathfrak{b} \wedge \bar{\mathfrak{a}} \underset{\vee\,\text{congr}}{\Rightarrow} \mathfrak{a} = \mathfrak{a} \wedge \underbrace{\bar{\mathfrak{b}} \vee \mathfrak{b}}_{= e}$$

$$\underset{\text{distr}}{=} \underbrace{\mathfrak{a} \wedge \bar{\mathfrak{b}}} \vee \underbrace{\mathfrak{a} \wedge \mathfrak{b}} \sim \underbrace{\mathfrak{b} \wedge \bar{\mathfrak{a}}} \vee \underbrace{\mathfrak{a} \wedge \mathfrak{b}} = \underbrace{\mathfrak{b} \wedge \bar{\mathfrak{a}}} \vee \underbrace{\mathfrak{b} \wedge \mathfrak{a}} \underset{\text{distr}}{=} \mathfrak{b} \wedge \underbrace{\bar{\mathfrak{a}} \vee \mathfrak{a}}_{= e} = \mathfrak{b} \Rightarrow \mathfrak{a} \sim \mathfrak{b}$$