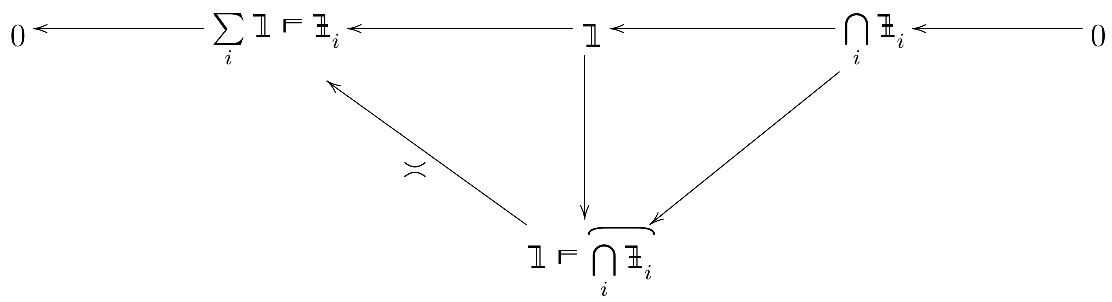
$$\mathbb{I}_i \triangleleft \mathbb{1} \in \triangledown\mathbb{Z} \text{ unit comm}$$

$$\bigwedge_{i<j} \mathbb{1} = \mathbb{I}_i + \mathbb{I}_j$$



$$\bigwedge_{i<j} \bigvee_{{}_j\mathbb{T}_i}^{\mathbb{I}_i} \bigvee_{{}_i\mathbb{T}_j}^{\mathbb{I}_j} {}_j\mathbb{T}_i + {}_i\mathbb{T}_j = 1$$

$$\prod_j^{\neq k} {}_k\mathbb{T}_j - 1 = \prod_j^{\neq k} {}_k\mathbb{T}_j - \prod_j^{\neq k} \overbrace{{}_k\mathbb{T}_j + {}_j\mathbb{T}_k}^{=1} \in \mathbb{I}_k$$

$$\bigwedge_i^{\neq k} \prod_j^{\neq i} {}_i\mathbb{T}_j = {}_k\mathbb{T}_j \prod_j^{\neq i:k} {}_i\mathbb{T}_j \in \mathbb{I}_k$$

$$\mathbb{T}^k \in \mathbb{1}$$

$$\mathbb{T} = \sum_i \mathbb{T}^i \prod_j^{\neq i} {}_i\mathbb{T}_j \Rightarrow \mathbb{T} - \mathbb{T}^k = \mathbb{T}^k \overbrace{\prod_j^{\neq k} {}_k\mathbb{T}_j - 1}^{\in \mathbb{I}_k} + \sum_i^{\neq k} \mathbb{T}^i \overbrace{\prod_j^{\neq i} {}_i\mathbb{T}_j}^{\in \mathbb{I}_k} \in \mathbb{I}_k$$