$$\acute{t}\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} \xleftarrow{\;\acute{t}|t\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\;} t\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} \xleftarrow{\;0|t\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\;} 0\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}$$

$$0\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} \xrightarrow{\;0|\acute{t}\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\;} \acute{t}\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}$$

$$\acute{t}|t\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\;-\infty|t\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} = -\infty|\acute{t}\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}$$

$$t\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} \xleftarrow[\;e^{t\mathcal{H}}e^{-t\underline{\mathcal{H}}}\;]{\;|t0t|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\;} t\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}$$

$$|t0t|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} = \overleftrightarrow{\exp\int_{d\tau}^{0|t}\bar{\mathcal{H}}_{\tau}}$$

$$\bar{\mathcal{H}}_{\tau} = e^{t\underline{\mathcal{H}}}\,\bar{\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} = e^{t\underline{\mathcal{H}}}\,\overbrace{\mathcal{H}-\underline{\mathcal{H}}}\,e^{-t\underline{\mathcal{H}}} = e^{t\underline{\mathcal{H}}}\mathcal{H}\,e^{-t\underline{\mathcal{H}}} - \underline{\mathcal{H}}$$

$$\frac{d}{dt}|t0t|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} = \frac{d}{dt}e^{t\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} = \mathcal{H}\,e^{t\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} - e^{t\mathcal{H}}\,\underline{\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} = e^{t\mathcal{H}}\,e^{-t\underline{\mathcal{H}}}\,\underbrace{e^{t\underline{\mathcal{H}}}\mathcal{H}e^{-t\underline{\mathcal{H}}}-\underline{\mathcal{H}}} = |t0t|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\,\bar{\mathcal{H}}_{t}$$

$$|000|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} = I \Longrightarrow |t0t|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R} = I + \int_{dt_1}^{0|t}|t_1 0 t_1|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\,\bar{\mathcal{H}}_{t_1} = I + \int_{dt_1}^{0|t}\overbrace{I + \int_{dt_2}^{0|t_1}|t_2 0 t_2|\times\mathbb{R}^{d}{\llcorner}^{\sharp}\,\mathbb{R}\bar{\mathcal{H}}_{t_2}}\,\bar{\mathcal{H}}_{t_1} = \cdots$$

$$= \sum_{n}^{\mathbb{N}}\int_{dt_1}^{0|t}\int_{dt_2}^{0|t_1}\cdots\int_{dt_n}^{0|t_{n-1}}\bar{\mathcal{H}}_{t_1}\cdots\bar{\mathcal{H}}_{t_n} = \sum_{n}^{\mathbb{N}}\int_{dt_1}^{t\geqslant t_1\geqslant\cdot\cdot}\cdots\int_{dt_n}^{\cdot\cdot\geqslant t_n\geqslant 0}\bar{\mathcal{H}}_{t_1}\cdots\bar{\mathcal{H}}_{t_n} = \sum_{n}^{\mathbb{N}}\frac{1}{n!}\int_{dt_1}^{0|t}\cdots\int_{dt_n}^{0|t}\overleftrightarrow{\bar{\mathcal{H}}_{t_1}\cdots\bar{\mathcal{H}}_{t_n}} = \overleftrightarrow{\exp\int_{d\tau}^{0|t}\bar{\mathcal{H}}_{\tau}}$$

$$|t\acute{t}|\times\mathbb{R}^{d}{}_{\llcorner}^{\sharp}\mathbb{R} = \overbrace{\exp\int_{d\tau}^{t|\acute{t}} \bar{\mathcal{H}}_\tau}$$

$$\bar{\mathcal{H}}_\tau = e^{t\underline{\mathcal{H}}}\,\bar{\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} = e^{t\underline{\mathcal{H}}}\,\overbrace{\mathcal{H}-\underline{\mathcal{H}}}\,e^{-t\underline{\mathcal{H}}} = e^{t\underline{\mathcal{H}}}\mathcal{H}\,e^{-t\underline{\mathcal{H}}} - \underline{\mathcal{H}}$$

$$\frac{d}{dt}|t\acute{t}|\times\mathbb{R}^{d}{}_{\llcorner}^{\sharp}\mathbb{R} = \frac{d}{dt}e^{t\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} = \mathcal{H}\,e^{t\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} - e^{t\mathcal{H}}\,\underline{\mathcal{H}}\,e^{-t\underline{\mathcal{H}}} = e^{t\mathcal{H}}\,e^{-t\underline{\mathcal{H}}}\,\underbrace{e^{t\underline{\mathcal{H}}}\mathcal{H}e^{-t\underline{\mathcal{H}}} - \underline{\mathcal{H}}} = |\acute{\lambda}|\times\mathbb{R}^{d}{}_{\llcorner}^{\sharp}\mathbb{R}\,\bar{\mathcal{H}}_t$$

$$|tt|\times\mathbb{R}^{d}{}_{\llcorner}^{\sharp}\mathbb{R} = I \Rightarrow |t\acute{t}|\times\mathbb{R}^{d}{}_{\llcorner}^{\sharp}\mathbb{R} = I + \int_{dt_1}^{t|\acute{t}} |t_1\acute{t}|\times\mathbb{R}^{d}{}_{\llcorner}^{\sharp}\mathbb{R}\,\bar{\mathcal{H}}_{t_1} = I + \int_{dt_1}^{t|\acute{t}} \overbrace{I + \int_{dt_2}^{t_1|\acute{t}} |t_2\acute{t}|\times\mathbb{R}^{d}{}_{\llcorner}^{\sharp}\mathbb{R}\bar{\mathcal{H}}_{t_2}}\,\bar{\mathcal{H}}_{t_1} = \cdots$$

$$= \sum_n^{\mathbb{N}} \int_{dt_1}^{t|\acute{t}} \int_{dt_2}^{t_1|\acute{t}} \cdots \int_{dt_n}^{t_{n-1}|\acute{t}} \bar{\mathcal{H}}_{t_n}\cdots\bar{\mathcal{H}}_{t_1} = \sum_n^{\mathbb{N}} \int_{dt_1}^{t\leqslant t_1\leqslant\cdot\cdot} \cdots \int_{dt_n}^{\cdot\cdot\leqslant t_n\leqslant\acute{t}} \bar{\mathcal{H}}_{t_n}\cdots\bar{\mathcal{H}}_{t_1} = \sum_n^{\mathbb{N}} \frac{1}{n!}\int_{dt_1}^{t|\acute{t}} \cdots \int_{dt_n}^{t|\acute{t}} \overbrace{\bar{\mathcal{H}}_{t_1}\cdots\bar{\mathcal{H}}_{t_n}} = \overbrace{\exp\int_{d\tau}^{t|\acute{t}} \bar{\mathcal{H}}_\tau}$$