

$$\frac{dy}{dx} = -\frac{x(1-a^2y^2)}{y(1-a^2x^2)}$$

$$\frac{xdx}{1-a^2x^2} + \frac{ydy}{1-a^2y^2} = 0$$

$$x^2 + y^2 = a^2 + a^2x^2y^2 \Rightarrow \underbrace{1-a^2x^2}\underbrace{1-a^2y^2} = 1 - a^4$$

$$\Rightarrow 0 = -2a^2 \overbrace{xdx(1-a^2y^2)} + \overbrace{ydy(1-a^2x^2)}$$

$$az = \frac{xv + yu}{1 + xyuv}; \quad aw = \frac{yv - xu}{1 - xyuv}$$

$$\underbrace{1-a^2z^2}\overbrace{1+xyuv}^2 = \underbrace{1-x^2v^2}\underbrace{1-y^2u^2}$$

$$\underbrace{1-az}\underbrace{1+xyuv} = \underbrace{1-xv}\underbrace{1-yu}$$

$$\underbrace{1+az}\underbrace{1+xyuv} = \underbrace{1+xv}\underbrace{1+yu}$$

$$a\frac{dz}{dx} = v\frac{1-a^2z^2}{1-x^2v^2}; \quad a\frac{dz}{dy} = u\frac{1-a^2z^2}{1-y^2u^2}$$

$$\text{LHS} = \frac{v\overbrace{1+xyuv} - \overbrace{xv+yu}yu}{\overbrace{1+xyuv}^2} = v\frac{1-y^2u^2}{\overbrace{1+xyuv}^2} = \text{RHS}$$

$$\text{LHS} = \frac{u\overbrace{1+xyuv} - \overbrace{xv+yu}xv}{\overbrace{1+xyuv}^2} = u\frac{1-x^2v^2}{\overbrace{1+xyuv}^2} = \text{RHS}$$

$$\overline{yv \underbrace{1 - a^2x^2} \underbrace{1 - y^2u^2} - xu \underbrace{1 - a^2y^2} \underbrace{1 - x^2v^2}} \underbrace{1 - xyuv} = \overline{yv - xu} \underbrace{1 - x^2v^2} \underbrace{1 - y^2u^2}$$

$$\begin{aligned} & \overline{\overbrace{1 - a^2x^2} yv \underbrace{1 - y^2u^2} - \underbrace{1 - a^2y^2} xu \underbrace{1 - x^2v^2}} \underbrace{1 - x^2y^2u^2v^2} \\ &= yv \underbrace{1 - x^2u^2} \underbrace{1 - x^2v^2} \underbrace{1 - y^2u^2} - xu \underbrace{1 - u^2y^2} \underbrace{1 - x^2v^2} \underbrace{1 - y^2v^2} \\ &= \underbrace{1 - x^2v^2} \underbrace{1 - y^2u^2} \overbrace{yv \underbrace{1 - x^2u^2} - xu \underbrace{1 - y^2v^2}} = \underbrace{1 - x^2v^2} \underbrace{1 - y^2u^2} \overbrace{yv - xu} \underbrace{1 + xyuv} \end{aligned}$$

$$\overbrace{1 - a^2x^2} y \frac{\partial z}{\partial x} = \overbrace{1 - a^2z^2} w: \quad \overbrace{1 - a^2u^2} v \frac{\partial z}{\partial u} = \overbrace{1 - a^2z^2} w$$

$$\begin{aligned} a \frac{\partial z}{\partial x} &= a \frac{dz}{dx} + a \frac{dz}{dy} \frac{dy}{dx} = v \frac{1 - a^2z^2}{1 - x^2v^2} - u \frac{1 - a^2z^2}{1 - y^2u^2} \frac{x}{y} \frac{1 - a^2y^2}{1 - a^2x^2} \\ a \frac{1 - a^2x^2}{1 - a^2z^2} y \frac{\partial z}{\partial x} &= vy \frac{1 - a^2x^2}{1 - x^2v^2} - ux \frac{1 - a^2y^2}{1 - y^2u^2} \\ &= \frac{yv \overbrace{1 - a^2x^2} \overbrace{1 - y^2u^2} - xu \overbrace{1 - a^2y^2} \overbrace{1 - x^2v^2}}{\underbrace{1 - x^2v^2} \underbrace{1 - y^2u^2}} = \frac{yv - xu}{1 - xyuv} = aw \end{aligned}$$