$$\boxed{x{:}\overset{i}{\mathsf{H}}{:}\,{}_{\mu}\overset{i}{\mathsf{H}}} \overset{\text{group}}{\underset{\text{inv}}{=}} \det {}^{x}\underline{\mathfrak{z}} \boxed{\overset{i}{\boxed{x{:}\mathsf{H}}}{:}{}^{x-1}_{\mu}\underline{\mathfrak{z}}^{\nu}\,\overbrace{{}_{\nu}\overset{i}{\boxed{x{:}\mathsf{H}}} + {}_{\nu}\overset{j}{\mathsf{H}}\underset{j}{\overset{i}{\boxed{x{:}\mathsf{H}}}}}^{x\mathfrak{z}}}$$

$$0 \overset{\text{Lie alg}}{\underset{\text{inv}}{=}} {}^{x}_{\mu}\underline{\mathfrak{z}}^{\mu} \boxed{x{:}\mathsf{H}{:}\mathsf{H}} + {}^{x}\mathfrak{z}^{\nu}\,{}_{\nu}\boxed{x{:}\mathsf{H}{:}\mathsf{H}} + \overset{i}{\boxed{x{:}\mathsf{H}}}\bullet \underset{i}{\boxed{x{:}\mathsf{H}{:}\mathsf{H}}} + \overbrace{{}_{\mu}\underline{\overset{i}{\boxed{x{:}\mathsf{H}}}} + {}_{\mu}\overset{j}{\mathsf{H}}\underset{j}{\overset{i}{\boxed{x{:}\mathsf{H}}}} - {}^{x}_{\mu}\underline{\mathfrak{z}}^{\nu}\,{}_{\nu}\overset{i}{\mathsf{H}}}\underset{i}{\boxed{x{:}\mathsf{H}{:}\mathsf{H}}}^{\mu}$$

$$\overset{i}{\boxed{x{:}\mathsf{H}}}\bullet = \frac{d}{dt}\overset{i}{\boxed{x{:}\mathsf{H}}}t$$

$${}^{x}\mathfrak{z} = \frac{d}{dt}\,{}^{x}\mathfrak{g} \Longrightarrow \frac{d}{dt}\det {}^{x}\underline{\mathfrak{g}}\cdot = \operatorname{tr}\frac{d}{dt}\,{}^{x}\underline{\mathfrak{g}}\cdot = \operatorname{tr}\,{}^{x}\underline{\mathfrak{z}}\cdot = {}^{x}_{\mu}\underline{\mathfrak{z}}^{\mu}$$

$$0 = \frac{d}{dt}\boxed{x{:}\mathsf{H}{:}\mathsf{H}} = \frac{d}{dt}\det {}^{x}\underline{\mathfrak{g}}\cdot \boxed{\overset{i}{\boxed{x{:}\mathsf{H}}}t{:}{}^{x-1}_{\mu}\underline{\mathfrak{g}}^{\nu}\,\overbrace{{}_{\nu}\overset{i}{\boxed{x{:}\mathsf{H}}}t + {}_{\nu}\overset{j}{\mathsf{H}}\underset{j}{\overset{i}{\boxed{x{:}\mathsf{H}}}}t}^{x\mathfrak{g}}}$$

$$= \frac{d}{dt}\det {}^{x}\underline{\mathfrak{g}}\cdot \boxed{x{:}\mathsf{H}{:}\mathsf{H}} + \frac{d}{dt}\boxed{\overset{i}{\boxed{x{:}\mathsf{H}}}t{:}{}^{x-1}_{\mu}\underline{\mathfrak{g}}^{\nu}\,\overbrace{{}_{\nu}\overset{i}{\boxed{x{:}\mathsf{H}}}t + {}_{\nu}\overset{j}{\mathsf{H}}\overset{i}{\boxed{x{:}\mathsf{H}}}t}^{x\mathfrak{g}}}$$

$$= {}^{x}_{\mu}\underline{\mathfrak{z}}^{\mu} \boxed{x{:}\mathsf{H}{:}\mathsf{H}} + {}^{x}\mathfrak{z}^{\nu}\,{}_{\nu}\boxed{x{:}\mathsf{H}{:}\mathsf{H}} + \overset{i}{\boxed{x{:}\mathsf{H}}}\bullet \underset{i}{\boxed{x{:}\mathsf{H}{:}\mathsf{H}}} + \overbrace{{}_{\mu}\overset{i}{\boxed{x{:}\mathsf{H}}}\bullet + {}_{\mu}\overset{j}{\mathsf{H}}\underset{j}{\overset{i}{\boxed{x{:}\mathsf{H}}}} - {}^{x}_{\mu}\underline{\mathfrak{z}}^{\nu}\,{}_{\nu}\overset{i}{\mathsf{H}}}\underset{i}{\boxed{x{:}\mathsf{H}{:}\mathsf{H}}}^{\mu} = \text{RHS}$$

$$x^{\nu}{:}\overset{i}{\mathsf{H}}{:}\,{}_{\mu}\overset{i}{\mathsf{H}} \in \mathbb{R}^{d}\times{}^{N}\mathbb{R}\times{}^{N}_{d}\mathbb{R} \xrightarrow[\text{el current}]{\boxed{\mathsf{H}{:}}\bullet^{\mu}} \mathbb{R} \ni \boxed{x{:}\mathsf{H}{:}\mathsf{H}}\bullet^{\mu}$$

$$\boxed{x{:}\mathsf{H}{:}\mathsf{H}}\bullet^{\mu} = {}^{x}\mathfrak{z}^{\mu}\boxed{x{:}\mathsf{H}{:}\mathsf{H}} + \underbrace{\overset{i}{\boxed{x{:}\mathsf{H}}}\bullet - {}^{x}\mathfrak{z}^{\nu}\,{}_{\nu}\overset{i}{\mathsf{H}}}\underset{i}{\boxed{x{:}\mathsf{H}{:}\mathsf{H}}}^{\mu} = {}^{x}\mathfrak{z}^{\nu}\,\overbrace{{}_{\nu}\delta^{\mu}\boxed{x{:}\mathsf{H}{:}\mathsf{H}} - {}_{\nu}\overset{i}{\mathsf{H}}\underset{i}{\boxed{x{:}\mathsf{H}{:}\mathsf{H}}}^{\mu}} + \overset{i}{\boxed{x{:}\mathsf{H}}}\bullet\underset{i}{\boxed{x{:}\mathsf{H}{:}\mathsf{H}}}^{\mu}$$

$$\mathsf{Y} \in {}^{\mathsf{h}}\underset{\infty}{\triangle}{}^{r}\mathbb{K}_{r}^{\mathbb{C}}$$

$$x\times^{r}\mathbb{K} \xleftarrow{\;{}^{x}\underset{\sim}{\Upsilon}\;} x\times^{r}\mathbb{K} \xleftarrow{\;{}^{x}\underset{\sim}{\Psi}\;} x\times^{r}\mathbb{K}$$

$$x\times^{r}\mathbb{K} \xleftarrow[\;{}^{x}\underset{\sim}{\Upsilon\Psi}\;]{} x\times^{r}\mathbb{K}$$

$${}^{x}\overbrace{\underset{\sim}{\Upsilon}\mathcal{H}}^{i} = {}^{x}\Upsilon^{i}{}_{j}\,\mathcal{H}^{j}$$

$$\begin{cases} {}^{x}\wr & = x \\ {}^{x}\underbrace{\mathcal{H}^{i}}_{\mathcal{H}} & = {}^{x}\Upsilon^{i}{}_{j}\,\mathcal{H}^{j} \end{cases}$$

$${}^{x}{}_{\mu}\wr^{-1\nu}_{-}\;\overbrace{{}^{x}\underbrace{\partial_{\nu}\mathcal{H}^{i}}_{\mathcal{H}} + {}^{x}\underbrace{\mathcal{H}^{i}\partial_{j}}_{\mathcal{H}}\;{}_{\nu}\mathcal{N}^{j}} = {}^{x}{}_{\mu}\Upsilon^{i}_{-j}\,\mathcal{H}^{j} + {}^{x}\Upsilon^{i}{}_{j}\;{}_{\mu}\mathcal{N}^{j}$$

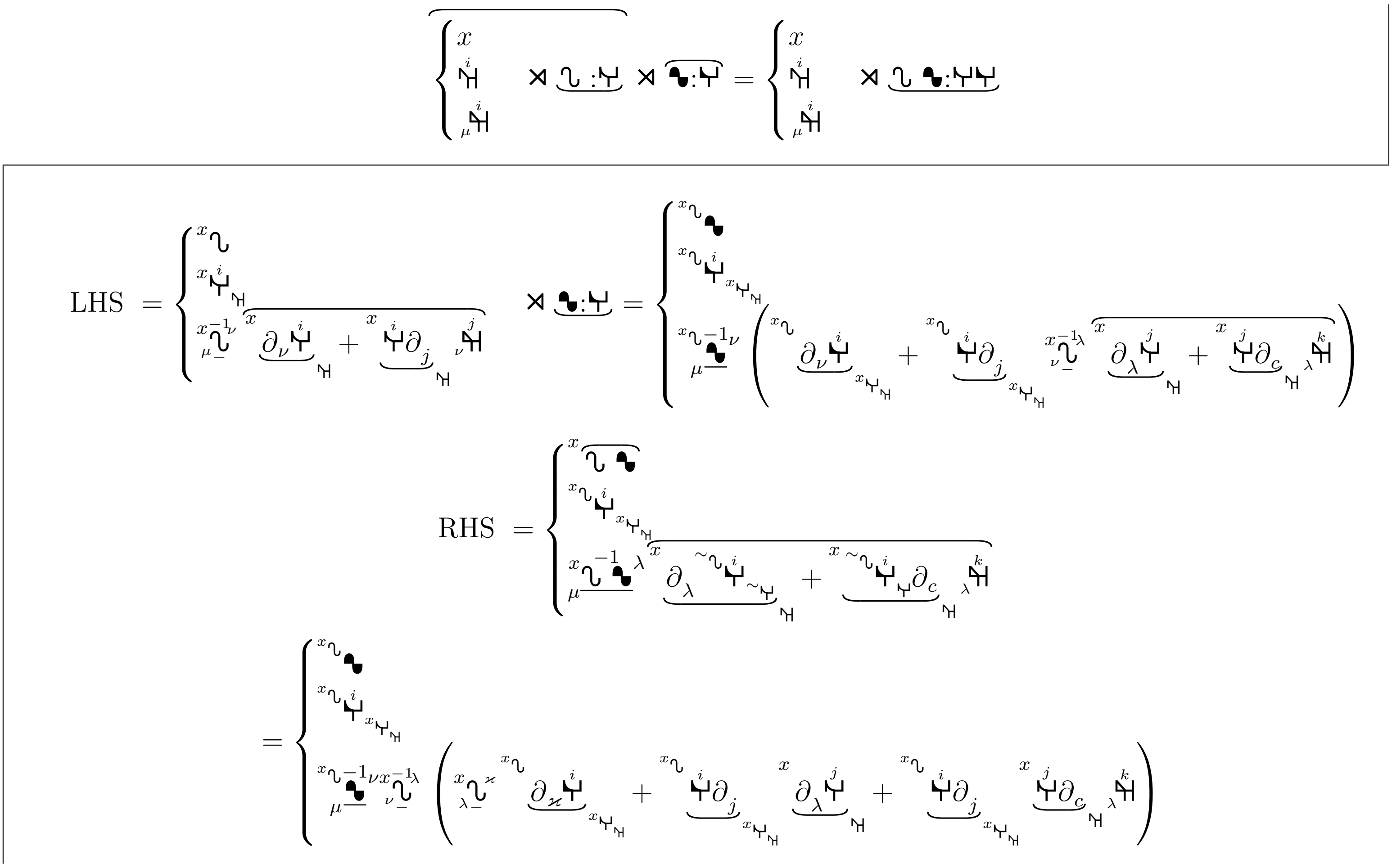
$${}^{x}\underbrace{\partial_{\nu}\mathcal{H}^{i}}_{\mathcal{H}} = {}^{x}{}_{\nu}\Upsilon^{i}_{-j}\,\mathcal{H}^{j}$$

$${}^{x}\underbrace{\mathcal{H}^{i}\partial_{j}}_{\mathcal{H}} = \frac{\partial\mathcal{H}^{i}}{\partial\mathcal{H}^{j}} = {}^{x}\Upsilon^{i}{}_{j}$$

$${}^{x}\underbrace{\mathcal{H}^{i}\partial_{j}}_{\mathcal{H}}\;{}_{\nu}\mathcal{N}^{j} = {}^{x}\Upsilon^{i}{}_{j}\;{}_{\nu}\mathcal{N}^{j}$$

$$\text{LHS} = {}_{\mu}\delta^{\nu}\;\underbrace{{}^{x}{}_{\nu}\Upsilon^{i}_{-j}\,\mathcal{H}^{j} + {}^{x}\Upsilon^{i}{}_{j}\;{}_{\nu}\mathcal{N}^{j}} = \text{RHS}$$

$$\begin{cases} x \\ \mathcal{H}^{i} \\ {}_{\mu}\mathcal{N}^{i} \end{cases} \ltimes \underbrace{\wr : \Upsilon} = \begin{cases} {}^{x}\wr \\ {}^{x}\Upsilon^{i}{}_{\mathcal{H}} \\ {}^{x}{}_{\mu}\wr^{-1\nu}_{-}\;\overbrace{{}^{x}\underbrace{\partial_{\nu}\Upsilon^{i}}_{\mathcal{H}} + {}^{x}\underbrace{\Upsilon^{i}\partial_{j}}_{\mathcal{H}}\;{}_{\nu}\mathcal{N}^{j}} \end{cases}$$


LHS
RHS