

$$x:\overset{i}{\mathbb{H}}:_{\mu}^i \underset{\text{inv}}{\equiv} \underset{\text{group}}{\equiv} \det \overset{x}{\mathfrak{L}} \boxed{\overset{i}{x:\mathbb{H}}:_{\mu}^{x^{-1}\nu} \overset{x_{\mathfrak{L}}}{\overbrace{\nu \overset{i}{x:\mathbb{H}} + \nu^j \overset{i}{\mathbb{H}} x:\mathbb{H}}}^j}$$

$$0 \underset{\text{inv}}{\equiv}^{\text{Lie alg}} \overset{x}{\mathfrak{A}}^{\mu} [x:\mathbb{H}:\mathbb{H}] + \overset{x}{\mathfrak{A}}^{\nu} [\nu x:\mathbb{H}:\mathbb{H}] + \overset{i}{[x:\mathbb{H}] \bullet [x:\mathbb{H}:\mathbb{H}]} + \overset{i}{[\mu \overset{i}{x:\mathbb{H}} + \mu^j \overset{i}{\mathbb{H}} x:\mathbb{H}]} - \overset{x}{\mathfrak{A}}^{\nu} \overset{i}{\mathfrak{A}}^i [x:\mathbb{H}:\mathbb{H}]^{\mu}$$

$$\begin{aligned} \overset{i}{[x:\mathbb{H}] \bullet} &= \frac{d}{dt} \overset{i}{[x:\mathbb{H}]} t \\ \overset{x}{\mathfrak{A}} = \frac{d}{dt} \overset{x}{\mathfrak{A}} &\Rightarrow \frac{d}{dt} \det \overset{x}{\mathfrak{A}} = \text{tr} \frac{d}{dt} \overset{x}{\mathfrak{A}} = \text{tr} \overset{x}{\mathfrak{A}} = \overset{x}{\mathfrak{A}}^{\mu} \\ 0 = \frac{d}{dt} [x:\mathbb{H}:\mathbb{H}] &= \frac{d}{dt} \det \overset{x}{\mathfrak{A}} \boxed{\overset{i}{[x:\mathbb{H}] t:_{\mu}^{x^{-1}\nu} \overset{i}{[x:\mathbb{H}] t} + \nu^j \overset{i}{\mathbb{H}} [x:\mathbb{H}] t}} \\ &= \frac{d}{dt} \det \overset{x}{\mathfrak{A}} [x:\mathbb{H}:\mathbb{H}] + \frac{d}{dt} \boxed{\overset{i}{[x:\mathbb{H}] t:_{\mu}^{x^{-1}\nu} \overset{i}{[x:\mathbb{H}] t} + \nu^j \overset{i}{\mathbb{H}} [x:\mathbb{H}] t}} \\ &= \overset{x}{\mathfrak{A}}^{\mu} [x:\mathbb{H}:\mathbb{H}] + \overset{x}{\mathfrak{A}}^{\nu} [\nu x:\mathbb{H}:\mathbb{H}] + \overset{i}{[x:\mathbb{H}] \bullet [x:\mathbb{H}:\mathbb{H}]} + \overset{i}{[\mu \overset{i}{x:\mathbb{H}} + \mu^j \overset{i}{\mathbb{H}} x:\mathbb{H}]} - \overset{x}{\mathfrak{A}}^{\nu} \overset{i}{\mathfrak{A}}^i [x:\mathbb{H}:\mathbb{H}]^{\mu} = \text{RHS} \end{aligned}$$

$$x^{\nu}: \overset{i}{\mathbb{H}}:_{\mu}^i \in \mathbb{R}^d \times {}^N \mathbb{R} \times {}_d^N \mathbb{R} \xrightarrow[\text{el current}]{\boxed{\mathbb{H}:}^{\mu} \bullet} \mathbb{R} \ni \boxed{x:\mathbb{H}:\mathbb{H}}^{\mu}$$

$$\boxed{x:\mathbb{H}:\mathbb{H}}^{\mu} = \overset{x}{\mathfrak{A}}^{\mu} [x:\mathbb{H}:\mathbb{H}] + \boxed{\overset{i}{[x:\mathbb{H}] \bullet} - \overset{x}{\mathfrak{A}}^{\nu} \overset{i}{\mathbb{H}} [x:\mathbb{H}:\mathbb{H}]^{\mu}} = \overset{x}{\mathfrak{A}}^{\nu} \overset{\nu}{\delta^{\mu}} [x:\mathbb{H}:\mathbb{H}] - \overset{i}{\mathbb{H}} [x:\mathbb{H}:\mathbb{H}]^{\mu} + \boxed{\overset{i}{[x:\mathbb{H}] \bullet} [x:\mathbb{H}:\mathbb{H}]^{\mu}}$$

$$\mathbb{H} \in \overset{\mathbb{H}}{\bigtriangleup}_{\infty} {}^r \mathbb{K}_r^C$$

$$\begin{array}{ccc}
& \overset{x}{\cancel{\text{H}}} & \\
x \times^r \mathbb{K} & \xleftarrow{\sim} & x \times^r \mathbb{K} & \xleftarrow{\sim} x \times^r \mathbb{K} \\
& \xleftarrow{\sim} & & \xleftarrow{\sim} \\
& \overset{x}{\cancel{\text{H}} \cancel{\text{H}}} & &
\end{array}$$

$$\overset{x}{\cancel{\text{H}} \cancel{\text{H}}} = \overset{x}{\cancel{\text{H}}}{}_j^j$$

$$\begin{cases} \overset{x}{\cancel{\text{H}}} = x \\ \overset{x}{\cancel{\text{H}}}{}_j^j = \overset{x}{\cancel{\text{H}}}{}_j^j \end{cases}$$

$$\overset{x}{\cancel{\text{H}}}{}_{\mu}^{\nu} \overbrace{\partial_{\nu} \overset{i}{\cancel{\text{H}}}}_{\mathbb{H}} + \overbrace{\overset{x}{\cancel{\text{H}}} \partial_j^i}^j \nu \cancel{\text{H}} = \overset{x}{\cancel{\text{H}}}{}_j^i \overset{j}{\cancel{\text{H}}} + \overset{x}{\cancel{\text{H}}}{}_j^i \overset{j}{\cancel{\text{H}}}$$

$$\begin{aligned}
& \overset{x}{\cancel{\text{H}}}{}_j^i \overset{j}{\cancel{\text{H}}} = \overset{x}{\cancel{\text{H}}}{}_j^i \overset{j}{\cancel{\text{H}}} \\
& \overset{x}{\cancel{\text{H}}} \partial_j^i \nu \cancel{\text{H}} = \frac{\partial \overset{i}{\cancel{\text{H}}}}{\partial \overset{j}{\cancel{\text{H}}}} = \overset{x}{\cancel{\text{H}}}{}_j^i \\
& \overset{x}{\cancel{\text{H}}} \partial_j^i \nu \cancel{\text{H}} = \overset{x}{\cancel{\text{H}}}{}_j^i \nu \cancel{\text{H}} \\
& \text{LHS} = {}_{\mu} \delta^{\nu} \underbrace{\overset{x}{\cancel{\text{H}}}{}_j^i \overset{j}{\cancel{\text{H}}} + \overset{x}{\cancel{\text{H}}}{}_j^i \nu \cancel{\text{H}}}_{\text{RHS}} = \text{RHS}
\end{aligned}$$

$$\begin{cases} \overset{x}{\cancel{\text{H}}} \\ \overset{x}{\cancel{\text{H}}}{}_j^i \end{cases} \times \cancel{\text{H}} = \begin{cases} \overset{x}{\cancel{\text{H}}} \\ \overset{x}{\cancel{\text{H}}}{}_j^i \end{cases} \overbrace{\overset{x}{\cancel{\text{H}}}{}_{\mu}^{\nu} \overbrace{\partial_{\nu} \overset{i}{\cancel{\text{H}}}}_{\mathbb{H}} + \overbrace{\overset{x}{\cancel{\text{H}}} \partial_j^i}^j \nu \cancel{\text{H}}}^j$$

$$\overbrace{\begin{cases} x \\ \underset{\mu}{\text{H}}^i \end{cases}} \times \underbrace{\text{U} : \text{H}}_{\text{H}} \times \overbrace{\text{U} : \text{H}}_{\text{H}} = \begin{cases} x \\ \underset{\mu}{\text{H}}^i \end{cases} \times \underbrace{\text{U} \text{U} : \text{H} \text{H}}_{\text{H}}$$

$$\text{LHS} = \begin{cases} x \text{U} \\ x \underset{\mu}{\text{H}}^i \\ x^{-1}_{\nu} \underset{\mu}{\text{U}}^i \partial_{\nu} \underset{\mu}{\text{H}} + x \underset{\mu}{\text{U}}^i \partial_j \underset{\mu}{\text{H}}^j \end{cases} \times \overbrace{\text{U} : \text{H}}_{\text{H}} = \begin{cases} x \text{U} \\ x \underset{\mu}{\text{H}}^i \\ x \underset{\mu}{\text{U}}^{-1} \nu \left(x \underset{\mu}{\text{U}}^i \partial_{\nu} \underset{\mu}{\text{H}} + x \underset{\mu}{\text{U}}^i \partial_j \underset{\mu}{\text{H}} x^{-1}_{\nu} \underset{\lambda}{\text{U}}^j \partial_{\lambda} \underset{\mu}{\text{H}} + x \underset{\mu}{\text{U}}^j \partial_c \underset{\lambda}{\text{H}}^k \right) \end{cases}$$

$$\text{RHS} = \begin{cases} x \text{U} \\ x \underset{\mu}{\text{H}}^i \\ x \underset{\mu}{\text{U}}^{-1} \nu \lambda \left(x \underset{\lambda}{\text{U}}^i \partial_{\nu} \underset{\lambda}{\text{H}} + x \underset{\mu}{\text{U}}^i \partial_c \underset{\lambda}{\text{H}}^k \right) \end{cases}$$

$$= \begin{cases} x \text{U} \\ x \underset{\mu}{\text{H}}^i \\ x \underset{\mu}{\text{U}}^{-1} \nu x^{-1}_{\lambda} \left(x \underset{\lambda}{\text{U}}^i \partial_{\nu} \underset{\lambda}{\text{H}} + x \underset{\mu}{\text{U}}^i \partial_j \underset{\mu}{\text{H}}^j + x \underset{\mu}{\text{U}}^j \partial_j \underset{\mu}{\text{H}}^i \right) \end{cases}$$