$$^x\boxed{\dashv:}\overset{\mu}{\bullet} = {}^x\mathfrak{X}^\mu \;{}^x\boxed{\dashv:} + \underbrace{{}^x_{\,i}\boxed{\dashv}\bullet - {}^x\mathfrak{X}^\nu{}^x_{\nu\underline{\ }}{}^i_{\dashv}}_{}\;{}^x\boxed{\dashv:}\overset{\mu}{}_i = {}^x\mathfrak{X}^\nu \overbrace{{}_\nu\delta^\mu{}^x\boxed{\dashv:} - {}^x_{\nu\underline{\ }}{}^i_{\dashv}{}^x\boxed{\dashv:}\overset{\mu}{}_i} + {}^x_i\boxed{\dashv}\bullet\;{}^x\boxed{\dashv:}\overset{\mu}{}_i$$

$$\boxed{\dashv:}\overset{\mu}{\bullet} = \mathfrak{X}^\mu\;\boxed{\dashv:} + \underbrace{{}^i\boxed{\dashv}\bullet - \mathfrak{X}^\nu{}_{\nu\underline{\ }}{}^i_{\dashv}}_{}\;\boxed{\dashv:}\overset{\mu}{}_i = \mathfrak{X}^\nu \overbrace{{}_\nu\delta^\mu\boxed{\dashv:} - {}_{\nu\underline{\ }}{}^i_{\dashv}\boxed{\dashv:}\overset{\mu}{}_i} + {}^i\boxed{\dashv}\bullet\;\boxed{\dashv:}\overset{\mu}{}_i$$

$$\boxed{\dashv:}\overset{\mu}{\bullet}{}_{\underline{\mu}} \overset{\text{conserved}}{\underset{\text{el current}}{=\!=}} 0$$

$$\text{LHS} = \overset{x}{}\mathfrak{X}^\nu{}_{\nu}\delta^\mu\boxed{\dashv:} - {}_{\nu\underline{\ }}{}^i_{\dashv}\boxed{\dashv:}\overset{\mu}{}_i + {}^i\boxed{\dashv}\bullet\;\boxed{\dashv:}\overset{\mu}{}_i{}_{\underline{\mu}}$$

$$= {}^x_{\mu\underline{\ }}\mathfrak{X}^\nu \underbrace{{}_\nu\delta^\mu{}^x\boxed{\dashv:} - {}^x_{\nu\underline{\ }}{}^i_{\dashv}{}^x\boxed{\dashv:}\overset{\mu}{}_i}_{} + {}^x\mathfrak{X}^\nu \underbrace{{}_\nu\delta^\mu\boxed{\dashv:} - {}_{\nu\underline{\ }}{}^i_{\dashv}\boxed{\dashv:}\overset{\mu}{}_i}_{\mu\;*} + \underbrace{{}^x_i\boxed{\dashv}\bullet\;{}^x\boxed{\dashv:}\overset{\mu}{}_i}_{\mu\;**} + \underbrace{{}^x_i\boxed{\dashv}\bullet\;{}^x\boxed{\dashv:}\overset{\mu}{}_i}_{\mu\;*\,*\,*}$$

$$\underset{\text{harm}}{=} {}^x_{\mu\underline{\ }}\mathfrak{X}^\nu \underbrace{{}_\nu\delta^\mu{}^x\boxed{\dashv:} - {}^x_{\nu\underline{\ }}{}^i_{\dashv}{}^x\boxed{\dashv:}\overset{\mu}{}_i}_{} + {}^x\mathfrak{X}^\nu \underbrace{{}^x_\nu\boxed{\dashv:}}_{*} + \underbrace{{}^x_i\boxed{\dashv}\bullet\;{}^x\boxed{\dashv:}}_{**}{}_i + \underbrace{{}^x_\mu\boxed{\dashv}\bullet + {}^x_{\mu\underline{\ }}{}^j_{\dashv}{}^x_j\boxed{\dashv}}_{*\,*\,*}{}^i\;{}^x\boxed{\dashv:}\overset{\mu}{}_i$$

$$= {}^x_{\mu\underline{\ }}\mathfrak{X}^\mu\;{}^x\boxed{\dashv:} + {}^x\mathfrak{X}^\nu\;{}^x_\nu\boxed{\dashv:} + {}^x_i\boxed{\dashv}\bullet\;{}^x\boxed{\dashv:}{}_i + \underbrace{{}^x_\mu\boxed{\dashv}\bullet + {}^x_{\mu\underline{\ }}{}^j_{\dashv}{}^x\boxed{\dashv}{}_j - {}^x_{\mu\underline{\ }}\mathfrak{X}^\nu{}^x_{\nu\underline{\ }}{}^i_{\dashv}}_{}\;{}^x\boxed{\dashv:}\overset{\mu}{}_i$$

$$= {}^x_{\mu\underline{\ }}\mathfrak{X}^\mu\;\boxed{{}^x\boxed{\dashv:}{}^x_{\underline{\dashv}}} + {}^x\mathfrak{X}^\nu{}_\nu\boxed{{}^x\boxed{\dashv:}{}^x_{\underline{\dashv}}} + {}^i\boxed{\dashv}\bullet\;\boxed{{}^x\boxed{\dashv:}{}^x_{\underline{\dashv}}}{}_i + \underbrace{{}^x_\mu\boxed{\dashv}\bullet + {}^x_{\mu\underline{\ }}{}^j_{\dashv}{}^x\boxed{\dashv} - {}^x_{\mu\underline{\ }}\mathfrak{X}^\nu{}^x_{\nu\underline{\ }}{}^i_{\dashv}}_{}\;\boxed{{}^x\boxed{\dashv:}{}^x_{\underline{\dashv}}}\overset{\mu}{}_i \overset{\text{Lie alg}}{\underset{\text{inv}}{=\!=}} 0$$

$$\text{conserved el charge } \partial_t \int\limits_S^{ds} \boxed{\dashv:}\overset{0}{\bullet} = 0$$

$$0 = \partial_\mu\boxed{\dashv:}\overset{\mu}{\bullet} = \mathfrak{d}\cdot\overline{\boxed{\dashv:}}\bullet + \partial_t\boxed{\dashv:}\overset{0}{\bullet} \Rightarrow 0 = \int\limits_S^{ds}\partial_\mu\boxed{\dashv:}\overset{\mu}{\bullet} = \underbrace{\int\limits_S^{ds}\mathfrak{d}\cdot\overline{\boxed{\dashv:}}\bullet}_{=0} + \int\limits_S^{ds}\partial_t\boxed{\dashv:}\overset{0}{\bullet} = \partial_t\int\limits_S^{ds}\boxed{\dashv:}\overset{0}{\bullet}$$