

$$\begin{array}{cccc} f & 0 & 0 & 0 \\ 0 & -f^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 s^2 \end{array}$$

$$\eta = \frac{\begin{array}{c|c|c|c} f_r & 0 & 0 & 0 \\ \hline 0 & -f^{-1} & 0 & 0 \\ \hline 0 & 0 & -r^2 & 0 \\ \hline 0 & 0 & 0 & -r^2 s^2 \end{array}}{=} = \frac{\begin{array}{c|c|c|c} f & 0 & 0 & 0 \\ \hline 0 & -f^{-1} & 0 & 0 \\ \hline 0 & 0 & -r^2 & 0 \\ \hline 0 & 0 & 0 & -r^2 s^2 \end{array}}{=}$$

$$\eta^{-1} = \frac{\begin{array}{c|c|c|c} f^{-1} & 0 & 0 & 0 \\ \hline 0 & -f & 0 & 0 \\ \hline 0 & 0 & -r^{-2} & 0 \\ \hline 0 & 0 & 0 & -r^{-2} s^{-2} \end{array}}{=}$$

$$\begin{array}{cccc} f & 0 & 0 & 0 \\ \partial_t & 0 & -f^{-1} & 0 \\ 0 & 0 & 0 & -r^2 \\ 0 & 0 & 0 & -r^2 s^2 \end{array}$$

$$\begin{array}{cccc} \emptyset & & & \\ \hline f & 0 & 0 & 0 \\ 0 & f/f^2 & 0 & 0 \\ 0 & 0 & -2r & 0 \\ 0 & 0 & 0 & -2r s^2 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2r^2 s c \\ \hline \emptyset & & & \end{array}$$

$$\eta \otimes \eta = \frac{\begin{array}{c|c|c|c} \partial_t & f & 0 & 0 \\ \hline \partial_r & 0 & -f^{-1} & 0 \\ \hline \partial_\vartheta & 0 & 0 & -r^2 \\ \hline \partial_\varphi & 0 & 0 & -r^2 s^2 \end{array}}{=}$$

$$\begin{array}{cccc} f & 0 & 0 & 0 \\ \partial_r & 0 & -f^{-1} & 0 \\ 0 & 0 & 0 & -r^2 \\ f & 0 & 0 & 0 \\ \partial_\vartheta & 0 & -f^{-1} & 0 \\ 0 & 0 & 0 & -r^2 \\ 0 & 0 & 0 & -r^2 s^2 \\ f & 0 & 0 & 0 \\ \partial_\varphi & 0 & -f^{-1} & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 s^2 \end{array}$$

0000



$$\begin{aligned}
\mathbf{a}_1 &= \frac{1}{2} \underbrace{\underline{\mathbf{u}} \mathbf{X} \mathbf{a}_1 - \underline{\mathbf{u}} \mathbf{X}^T \mathbf{a}_1 + \underline{\mathbf{u}} \mathbf{X}^t \mathbf{a}_1}_{\mathbf{a}_1^{-1}} = \\
& \begin{array}{cccc|cccc}
0 & -\underline{f}/2 & 0 & 0 & & & & \\
\underline{f}/2 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & 0 & & & & \\
\hline
\underline{f}/2 & 0 & 0 & 0 & & & & \\
0 & \underline{f}/2 \underline{f}^2 & 0 & 0 & & & & \\
0 & 0 & -r & 0 & & & & \\
0 & 0 & 0 & -rs^2 & & & & \\
\hline
0 & 0 & 0 & 0 & & & & \\
0 & 0 & -r & 0 & & & & \\
0 & r & 0 & 0 & & & & \\
0 & 0 & 0 & -r^2 sc & & & & \\
\hline
0 & 0 & 0 & 0 & & & & \\
0 & 0 & 0 & -rs^2 & & & & \\
0 & 0 & 0 & -r^2 sc & & & & \\
0 & rs^2 & r^2 sc & 0 & & & & \\
\hline
0 & -\underline{f}/2 & 0 & 0 & \underline{f}^{-1} & 0 & 0 & 0 & 0 & \underline{f}\underline{f}/2 & 0 & 0 \\
\underline{f}/2 & 0 & 0 & 0 & 0 & -\underline{f} & 0 & 0 & \underline{f}/2 \underline{f} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -r^{-2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -r^{-2} s^{-2} & 0 & 0 & 0 & 0 \\
\hline
\underline{f}/2 & 0 & 0 & 0 & \underline{f}^{-1} & 0 & 0 & 0 & \underline{f}/2 \underline{f} & 0 & 0 & 0 \\
0 & \underline{f}/2 \underline{f}^2 & 0 & 0 & 0 & -\underline{f} & 0 & 0 & 0 & -\underline{f}/2 \underline{f} & 0 & 0 \\
0 & 0 & -r & 0 & 0 & 0 & -r^{-2} & 0 & 0 & 0 & r^{-1} & 0 \\
0 & 0 & 0 & -rs^2 & 0 & 0 & 0 & -r^{-2} s^{-2} & 0 & 0 & 0 & r^{-1} \\
\hline
0 & 0 & 0 & 0 & \underline{f}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -r & 0 & 0 & -\underline{f} & 0 & 0 & 0 & 0 & r^{-1} & 0 \\
0 & r & 0 & 0 & 0 & 0 & -r^{-2} & 0 & 0 & -r \underline{f} & 0 & 0 \\
0 & 0 & 0 & -r^2 sc & 0 & 0 & 0 & -r^{-2} s^{-2} & 0 & 0 & 0 & c/s \\
\hline
0 & 0 & 0 & 0 & \underline{f}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -rs^2 & 0 & -\underline{f} & 0 & 0 & 0 & 0 & 0 & r^{-1} \\
0 & 0 & 0 & -r^2 sc & 0 & 0 & -r^{-2} & 0 & 0 & 0 & 0 & c/s \\
0 & rs^2 & r^2 sc & 0 & 0 & 0 & 0 & -r^{-2} s^{-2} & 0 & -r \underline{f} s^2 & -sc & 0
\end{array} =
\end{aligned}$$

$$\underline{\mathcal{L}} \underline{\Sigma} \underline{\mathcal{P}}_1^\# =$$

$\emptyset$				$\emptyset$	$\emptyset$				$\emptyset$			
0	$\underline{ff/2}$	0	0	*	0	0	0	0	0	0	0	0
$\underline{f/2f}$	0	0	0		0	0	$-r^{-2}$	0	0	0	0	$-r^{-2}$
0	0	0	0		0	$-f - r\underline{f}$	0	0	0	0	0	0
0	0	0	0		0	0	0	0	0	$\underline{-f + r\underline{f} s^2}$	0	0
$\emptyset$				$\emptyset$	*				0	0	0	0
$\emptyset$				$\emptyset$	$\emptyset$				0	0	0	0
$\emptyset$				$\emptyset$	$\emptyset$				0	0	0	$-s^{-2}$
$\emptyset$				$\emptyset$	$\emptyset$				0	$-2rfsc$	$s^2 - c^2$	0
$\emptyset$				$\emptyset$	$\emptyset$				$\emptyset$			



$$\underline{1} \underline{\alpha} \underline{a}_1^\# - \underline{a}_1 \underline{\alpha} \underline{a}_1^\# =$$

*	$\begin{matrix} 0 & \underline{f}^2/4 & 0 & 0 \\ -\underline{f}^2/4\bar{f}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & -\underline{f}\bar{f}/2r & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & -\underline{f}\bar{f}/2r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$
$\begin{matrix} 0 & \frac{\underline{f}\bar{f}}{2} + \frac{\bar{f}^2}{4} & 0 & 0 \\ \frac{\underline{f}}{2\bar{f}} - \frac{\bar{f}^2}{4\bar{f}^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$	*	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\underline{f}}{2r\bar{f}} - \frac{1}{r^2} & 0 \\ 0 & -r\bar{f} & 0 & 0 \\ 0 & 0 & 0 & -c/rs \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\underline{f}}{2r\bar{f}} - \frac{1}{r^2} \\ 0 & 0 & 0 & -c/rs \\ 0 & -r\bar{f}s^2 & sc/r & 0 \end{matrix}$
$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ rf/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -r^{-2} & 0 \\ 0 & -r\bar{f}/2 & 0 & 0 \\ 0 & 0 & 0 & -c/rs \end{matrix}$	*	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c/rs \\ 0 & 0 & 0 & \underline{f}-s^{-2} \\ 0 & -r\bar{f}sc & s^2 & 0 \end{matrix}$
$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ rf\bar{s}^2/2 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -r^{-2} \\ 0 & 0 & 0 & -c/rs \\ 0 & -r\bar{f}\bar{s}^2/2 & sc/r & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c/rs \\ 0 & 0 & 0 & -c^2/s^2 \\ 0 & -r\bar{f}sc & \bar{f}s^2 & 0 \end{matrix}$	*

$$\mathbf{a}_1 = \underline{1} \underline{x} \overset{\#}{\mathbf{a}}_1 - \underline{1} \overset{T}{\underline{x}} \overset{\#}{\mathbf{a}}_1 - \underbrace{\mathbf{a}_1 \underline{x} \overset{\#}{\mathbf{a}}_1 - \mathbf{a}_1 \overset{T}{\underline{x}} \overset{\#}{\mathbf{a}}_1}_T = \underline{1} \underline{x} \overset{\#}{\mathbf{a}}_1 - \mathbf{a}_1 \underline{x} \overset{\#}{\mathbf{a}}_1 - \overbrace{\underline{1} \underline{x} \overset{\#}{\mathbf{a}}_1 - \mathbf{a}_1 \underline{x} \overset{\#}{\mathbf{a}}_1} =$$

	$\emptyset$	$\begin{array}{cccc} 0 & -\underline{f}f & 0 & 0 \\ -\underline{f}/f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$	$\begin{array}{cccc} 0 & 0 & -\underline{f}f/r & 0 \\ 0 & 0 & 0 & 0 \\ -rf & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$	$\begin{array}{cccc} 0 & 0 & 0 & -\underline{f}f/r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -rf s^2 & 0 & 0 & 0 \end{array}$
$\frac{1}{2}$	$\begin{array}{cccc} 0 & \underline{f}f & 0 & 0 \\ \underline{f}/f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$	$\emptyset$	$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{f}/rf & 0 \\ 0 & -rf & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$	$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{f}/rf \\ 0 & 0 & 0 & 0 \\ 0 & -rf s^2 & 0 & 0 \end{array}$
$\frac{1}{2}$	$\begin{array}{cccc} 0 & 0 & \underline{f}f/r & 0 \\ 0 & 0 & 0 & 0 \\ rf & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$	$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & -\underline{f}/rf & 0 \\ 0 & rf & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}$	$\emptyset$	$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{2f-1} \\ 0 & 0 & \underline{21-f} s^2 & 0 \end{array}$
	$\begin{array}{cccc} 0 & 0 & 0 & \underline{f}f/r \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ rf s^2 & 0 & 0 & 0 \end{array}$	$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\underline{f}/rf \\ 0 & 0 & 0 & 0 \\ 0 & rf s^2 & 0 & 0 \end{array}$	$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{21-f} \\ 0 & 0 & \underline{2f-1} s^2 & 0 \end{array}$	$\emptyset$

