$$D_2 + \omega_2 = \begin{array}{c|c} (\daleth + \omega) \mathbin{⧗} 1_R & \varkappa \Gamma (\varphi + \varepsilon)^* \mathbin{⧗} e^* \\ \hline (\varphi + \varepsilon) \Gamma \mathbin{⧗} e & \left(\daleth + \omega/2 + \Omega_0\right) \mathbin{⧗} 1_L \end{array}$$

$$4\mathcal{L}\left(\omega_2 : \psi_2\right) = 2\psi_2 \mathbin{\underset{\mathbb{R}}{\bar{\curlywedge}}} i \left(D_2 + \omega_2\right) \psi_2 = \psi_2 \mathbin{\bar{\curlywedge}} i \left(D_2 + \omega_2\right) \psi_2 + \psi_2 \mathbin{\bar{\curlywedge}} i \left(D_2 + \omega_2\right) \psi_2$$

$$= \psi_2 \mathbin{\bar{\curlywedge}} i \underbrace{D_2 + \omega_2} \psi_2 + i \underbrace{D_2 + \omega_2} \psi_2 \mathbin{\bar{\curlywedge}} \psi_2 = i \left(\psi_2 \mathbin{\bar{\curlywedge}} \underbrace{D_2 + \omega_2} \psi_2 - \underbrace{D_2 + \omega_2} \psi_2 \mathbin{\bar{\curlywedge}} \psi_2\right)$$

$$\psi_2 \mathbin{\bar{\curlywedge}} \psi_2' = \psi_2^* \gamma^0 \psi_2' = \psi_2^* \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} \mathbin{⧗} \psi_2'$$

$$\Re i \begin{bmatrix} \psi \mathbin{⧗} e_R \\ \Psi \mathbin{⧗} \ell_L \end{bmatrix} \mathbin{\bar{\curlywedge}} \left(D_2 + \omega_2\right) \begin{bmatrix} \psi \mathbin{⧗} e_R \\ \Psi \mathbin{⧗} \ell_L \end{bmatrix} =$$

$$2\Re i \Psi \mathbin{\bar{\curlywedge}} \underbrace{\varphi + \varepsilon} \Gamma \psi \ell_L^* e e_R + \psi \mathbin{\bar{\curlywedge}} \underbrace{\daleth + \omega} \psi e_R^* e_R + \Psi \mathbin{\bar{\curlywedge}} \underbrace{\daleth + \omega/2 + \Omega_0} \Psi \ell_L^* \ell_L$$

$$\text{LHS} = \Re i \begin{bmatrix} \psi \mathbin{⧗} e_R \\ \Psi \mathbin{⧗} \ell_L \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} \underbrace{\daleth + \omega} \psi \mathbin{⧗} e_R + \varkappa \Gamma \overbrace{\varphi + \varepsilon}^* \Psi \mathbin{⧗} e^* \ell_L \\ \underbrace{\varphi + \varepsilon} \Gamma \psi \mathbin{⧗} e e_R + \underbrace{\daleth + \omega/2 + \Omega_0} \Psi \mathbin{⧗} \ell_L \end{bmatrix}$$

$$= \Re i \left(\psi \mathbin{\bar{\curlywedge}} \underbrace{\daleth + \omega} \psi e_R^* e_R + \varkappa \psi \mathbin{\bar{\curlywedge}} \Gamma \overbrace{\varphi + \varepsilon}^* \Psi e_R^* e^* \ell_L + \Psi \mathbin{\bar{\curlywedge}} \underbrace{\varphi + \varepsilon} \Gamma \psi \ell_L^* e e_R + \Psi \mathbin{\bar{\curlywedge}} \underbrace{\daleth + \omega/2 + \Omega_0} \Psi \ell_L^* \ell_L\right) = \text{RHS}$$

$$\daleth^* = -\daleth : \quad \omega^* = \bar{\omega} = -\omega : \quad \Omega_0^* = \Omega_0^* = -\Omega_0 : \quad \Gamma^* = -\varkappa \Gamma$$

$$\begin{array}{c|c} 0 & A \\ \hline -A^* & 0 \end{array} \begin{bmatrix} \psi \\ 0 \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} \psi \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -A^* \psi \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} \begin{bmatrix} \psi \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -A^* \psi \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} 0 \\ \psi \end{bmatrix} = -A^* \psi \mathbin{\bar{\curlywedge}} \psi$$

$$\begin{array}{c|c} 0 & A \\ \hline -A^* & 0 \end{array} \begin{bmatrix} 0 \\ \Psi \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} 0 \\ \Psi \end{bmatrix} = \begin{bmatrix} A\Psi \\ 0 \end{bmatrix} \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} \begin{bmatrix} 0 \\ \Psi \end{bmatrix} = \begin{bmatrix} A\Psi \\ 0 \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} \Psi \\ 0 \end{bmatrix} = A\Psi \mathbin{\bar{\curlywedge}} \Psi$$

$$\underbrace{\varphi + \varepsilon} \Gamma \begin{bmatrix} \psi \\ 0 \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} 0 \\ \Psi \end{bmatrix} = \underbrace{\varphi + \varepsilon} \begin{bmatrix} \psi \\ 0 \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} 0 \\ \Psi \end{bmatrix} = \underbrace{\varphi + \varepsilon} \begin{bmatrix} \psi \\ 0 \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} \begin{bmatrix} 0 \\ \Psi \end{bmatrix} = \underbrace{\varphi + \varepsilon} \begin{bmatrix} \psi \\ 0 \end{bmatrix} \mathbin{\bar{\curlywedge}} \begin{bmatrix} \Psi \\ 0 \end{bmatrix} = \underbrace{\varphi + \varepsilon} \psi \mathbin{\bar{\curlywedge}} \Psi$$