

$$\begin{aligned}
\text{left Weyl spinors } \frac{1}{2}:0 & \begin{cases} {}^2\mathbb{C} \ni \underline{\Psi} = \left(\begin{smallmatrix} A \\ \Psi \end{smallmatrix} \right) = \psi_L = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \left(\psi_A \right) & \overline{\underline{\Psi}}^A = {}^A \underline{\Psi}_B \\ \mathbb{C}_2 \ni \underline{\Psi} = \left(\begin{smallmatrix} \Psi_A \end{smallmatrix} \right) = \psi_R^* = \begin{bmatrix} \psi^1 & \psi^2 \end{bmatrix} = \left(\psi^A \right) & \underline{\Psi}^A = \Psi_B \overline{\underline{\Psi}}^B \end{cases} \\
\text{right Weyl spinors } 0:\frac{1}{2} & \begin{cases} {}^2\mathbb{C} \ni \underline{\Psi}^* = \left(\begin{smallmatrix} \Psi^* \end{smallmatrix} \right) = \psi_R = \begin{bmatrix} \psi^{*1} \\ \psi^{*2} \end{bmatrix} = \left(\psi^{*A} \right) & \overline{\underline{\Psi}^*}^A = {}^A \underline{\Psi}^* \overline{\underline{\Psi}}^B \\ \mathbb{C}_2 \ni \underline{\Psi}^* = \left(\begin{smallmatrix} \Psi^* \end{smallmatrix} \right) = \psi_L^* = \begin{bmatrix} \psi^*_1 & \psi^*_2 \end{bmatrix} = \left(\psi^*_A \right) & \underline{\Psi}^* = \Psi^*_A \overline{\underline{\Psi}^*}^B \end{cases}
\end{aligned}$$

$$\text{Dirac } \frac{1}{2}:0 \oplus 0:\frac{1}{2} \begin{cases} \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} = \begin{bmatrix} \underline{\Psi}^A \\ \underline{\Psi}_A \end{bmatrix} = \psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi^{*1} \\ \psi^{*2} \end{bmatrix} = \begin{bmatrix} \psi_A \\ \psi^*_A \end{bmatrix} \\ \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix}^\# = \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix}^* \frac{0}{1} \Big| \frac{1}{0} = \begin{bmatrix} \underline{\Psi} & \underline{\Psi}^* \end{bmatrix} = \begin{bmatrix} \underline{\Psi}_A & \underline{\Psi}^*_A \end{bmatrix} = \psi = \psi \gamma_0 = \begin{bmatrix} \psi_R & \psi_L \end{bmatrix} = \left(\psi^1 \psi^2 \psi^*_1 \psi^*_2 \right) = \begin{bmatrix} \psi^A & \psi^*_A \end{bmatrix} \end{cases}$$

$$\text{Majorana spinors } \frac{1}{2}:0 \oplus_{\mathbb{R}} 0:\frac{1}{2} \begin{cases} \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} = \Psi_R = -i\sigma^2 \overline{\underline{\Psi}}_L = \frac{0}{1} \Big| \frac{-1}{0} \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} = \begin{bmatrix} -\underline{\Psi} \\ \underline{\Psi} \end{bmatrix} \\ \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ -\underline{\Psi} \\ \underline{\Psi} \end{bmatrix} = \begin{bmatrix} \underline{\Psi} \\ -i^B \sigma^2 \overline{\underline{\Psi}}^A \end{bmatrix} \end{cases}$$

$$2^N \ni A \subset N$$

$$\text{dof } \begin{bmatrix} \underline{\Psi}^A \\ \underline{\Psi}_A \end{bmatrix} : \begin{bmatrix} \underline{\Psi}^A \\ \underline{\Psi}_A \end{bmatrix} \in {}^{2^N} \mathbb{K} \times {}^{2^N}_d \mathbb{K}$$

$$\begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} : \underline{\Psi} = \begin{bmatrix} \underline{\Psi}^A \\ \underline{\Psi}_A \end{bmatrix} = \begin{bmatrix} \underline{\Psi}^A \\ \underline{\Psi}_A \end{bmatrix}$$

$$= \det \underline{\Psi}^m \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix}^\# \underline{\Psi}^m \gamma^m \left(\begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} - \frac{1}{4} \omega^{\mu\nu} \gamma_{\mu\nu} \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} \right) = \begin{bmatrix} \underline{\Psi} & \underline{\Psi}^* \end{bmatrix} \frac{0}{\tilde{\sigma}^\mu} \Big| \frac{\sigma^\mu}{0} \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix} - m \begin{bmatrix} \underline{\Psi} & \underline{\Psi}^* \end{bmatrix} \begin{bmatrix} \underline{\Psi} \\ \underline{\Psi}^* \end{bmatrix}$$

$$= \underline{\Psi} \sigma^\mu \underline{\Psi}^* + \underline{\Psi}^* \tilde{\sigma}^\mu \underline{\Psi} - m \left(\underline{\Psi} \underline{\Psi} + \underline{\Psi}^* \underline{\Psi}^* \right) = \underline{\Psi}_A^A \sigma_B^\mu \underline{\Psi}^*_B + \underline{\Psi}^*_A \tilde{\sigma}^\mu \underline{\Psi}^B - m \left(\underline{\Psi}_A \underline{\Psi}^A + \underline{\Psi}^*_A \underline{\Psi}^*_A \right)$$

$$\begin{aligned}
&= \mathfrak{N}_A \left({}^A\sigma_{B^\mu} \mathfrak{N}_B^* - m \mathfrak{N}_A^* \right) + \mathfrak{N}_A^* \left({}^A\tilde{\sigma}_{B^\mu} \mathfrak{N}_B^* - m \mathfrak{N}_A^* \right) \\
= \psi^\# \gamma^\mu{}_\mu \Psi - m \psi^\# \psi &= \psi_R^* \sigma^\mu{}_\mu \Psi_R + \psi_L^* \sigma^2 \tilde{\sigma}^\mu \sigma^2{}_\mu \Psi_L - m \left(\psi_R^* \psi_L + \psi_L^* \psi_R \right) \\
&= \psi^A \sigma^\mu{}_{A\dot{B}} \Psi^{\dot{B}} + \psi_{\dot{A}}^* \overbrace{\sigma^2 \tilde{\sigma}^\mu \sigma^2}^{\dot{A}B}{}_\mu \Psi_B - m \left(\psi^A \psi_A + \psi_{\dot{A}}^* \psi^{\dot{A}} \right)
\end{aligned}$$