

$$\begin{aligned} \overline{d + \bar{1}}_1 &= \mathbb{1}^\mu \otimes \overline{d + \bar{1}}_{\mu\bar{1}} = \mathbb{1} \otimes \overline{d + \bar{1}}_{\bar{1}} \\ -d^* \mathbb{1} &= \sum_j \eta^{jj} \overline{1}_{j\bar{1}} \overline{1}_{\bar{1}} = \sum_j \eta^{jj} \overline{1}_{j\bar{1}} \otimes \overline{1}_{\bar{1}} - \overline{1}_{j\bar{1}} \overline{1}_{\bar{1}} \end{aligned}$$

$$\begin{aligned} \overline{1 \otimes d + \text{RHS}} \mathbb{1}^N &= \sum_j \eta^{jj} \overline{1}_{j\bar{1}} \overline{1}_{\bar{1}} \otimes \mathbb{1} + \overline{1}_{j\bar{1}} \overline{1}_{\bar{1}} \otimes \mathbb{1} \mathbb{1}^N = \sum_j \eta^{jj} \overline{1}_{j\bar{1}} \overline{1}_{\bar{1}} \otimes \mathbb{1}^N \\ &= \sum_j \mathbb{1}^j \otimes \overline{1}_{j\bar{1}} \otimes \mathbb{1}^N = \sum_j \overline{1}_{j\bar{1}} \otimes \mathbb{1}^j \otimes \mathbb{1}^N - \overline{1}_{j\bar{1}} \otimes \mathbb{1}^j \otimes \mathbb{1}^N \mathbb{1}^N \\ &= \sum_j \overline{1}_{j\bar{1}} \otimes \mathbb{1}^j \otimes \mathbb{1}^N \mathbb{1}^N - \overline{1}_{j\bar{1}} \otimes \mathbb{1}^j \otimes \mathbb{1}^N \\ &= \sum_j \overline{1}_{j\bar{1}} \otimes \mathbb{1}^j \otimes \mathbb{1}^N - \overline{1}_{j\bar{1}} \otimes \mathbb{1}^j \otimes \mathbb{1}^N = \sum_j \mathbb{1}^j \otimes \overline{1}_{j\bar{1}} \otimes \mathbb{1}^N = d \otimes \mathbb{1}^N \neq 0 \\ \Leftrightarrow 0 &= \overline{1}_{j\bar{1}} \otimes \mathbb{1}^N \otimes \mathbb{1}^N = 2 \overline{1}_{j\bar{1}} \otimes \mathbb{1}^N \otimes \mathbb{1}^N \Rightarrow \overline{1}_{j\bar{1}} \otimes \mathbb{1}^N = 0 \Rightarrow \overline{1}_{j\bar{1}} \otimes \mathbb{1}^N = 0 \end{aligned}$$