

$$\begin{aligned}
\bar{A}_i &= dA_i - A_i \times A_i \\
\bar{A}_\mu^\nu &= dA_\mu^\nu - A_\mu^\lambda \times A_\lambda^\nu \\
\flat\flat \bar{A}_\mu^\nu &= \flat\flat (dA_\mu^\nu) - \flat A_\mu^\lambda \left(\flat A_\lambda^\nu \right) + \flat A_\mu^\lambda \left(\flat A_\lambda^\nu \right) \\
&= \flat\flat (dA_\mu^\nu) - \flat\flat (A_\mu^\lambda A_\lambda^\nu) - \flat A_\mu^\lambda \left(\flat A_\lambda^\nu \right) + \flat A_\mu^\lambda \left(\flat A_\lambda^\nu \right) \\
&\quad d + \nabla
\end{aligned}$$

$$\begin{aligned}
\bar{A}_i^j &= \gamma_\mu^i \bar{A}_\mu^j \\
\mathcal{A}^i &= \gamma_\mu^i \mu \mathcal{A}^i = dx^\mu \mu \mathcal{A}^i \in \widetilde{\mathbb{R}}^n \triangleleft_n \mathbb{R} \\
\bar{A}_\mu^j &= \mu \mathcal{A}_\mu^j = \mu \nabla_\mu^j = i_\mu \Gamma^j \\
2 \mu \mathcal{A}_i \nabla_{jk}^j \mathcal{A} &= \mu \mathcal{A}_i | d\mathcal{A}_{jk} + \mathcal{A}_i | d\mu_{jk} - \mathcal{A}_i | d\mu_{jk} \\
&= \mu \partial_{ik} \mathcal{A} + i \partial_{\mu k} \mathcal{A} - k \partial_{i\mu} \mathcal{A} = 2_{ki\mu} \Gamma = 2_{i\mu} \Gamma_{jk}^j \mathcal{A} \Leftarrow \alpha\beta \Gamma^\mu = g^{\mu\nu} \nu\alpha\beta \Gamma \\
\bar{A}_i^j &= \gamma_\mu^i \mathcal{A}_\mu^j \\
\bar{A}_{\mu\nu}^j &= \mu \mathcal{A}_\mu^j \mathcal{A}_\nu^j
\end{aligned}$$

$$\bar{A}_{\mu\nu}^j = \mu \partial_{i\nu} \bar{A}_i^j - \nu \partial_{i\mu} \bar{A}_i^j - \bar{A}_i^k \bar{A}_{k\nu}^j + \bar{A}_i^k \bar{A}_{\mu k}^j = i_{\mu\nu} R^j$$

$$\begin{aligned}
\mu \mathcal{A}_\nu \mathcal{A}_\mu | \bar{A}_i^j &= \mu \mathcal{A}_\nu \mathcal{A}_\mu | \overline{d\bar{A}_i^j - \nabla_i^k \bar{A}_k^j} = \\
\mu \mathcal{A}_\nu \underbrace{d\mathcal{A}_\mu | \bar{A}_i^j}_{\mu \mathcal{A}_\mu^j} - \nu \mathcal{A}_\mu \underbrace{d\mathcal{A}_\mu | \bar{A}_i^j}_{\mu \mathcal{A}_\mu^j} - \underbrace{\mu \mathcal{A}_\mu^k}_{\mu \mathcal{A}_\mu^k} \underbrace{\nu \mathcal{A}_\mu^j}_{\nu \mathcal{A}_\mu^j} + \underbrace{\nu \mathcal{A}_\mu^k}_{\nu \mathcal{A}_\mu^k} \underbrace{\mu \mathcal{A}_\mu^j}_{\mu \mathcal{A}_\mu^j} &= \mu \partial_{i\nu} \bar{A}_i^j - \nu \partial_{i\mu} \bar{A}_i^j - \bar{A}_i^k \bar{A}_{k\nu}^j + \bar{A}_i^k \bar{A}_{\mu k}^j
\end{aligned}$$

$$\bar{A}_j^i \mathcal{A}_{i\ell} \gamma_j^j \mathcal{A}_\ell = \bar{A}_j^i \bar{A}_j^{\mu j} \text{ scal curv}$$

$$\begin{aligned}
2 \text{ LHS} &= \mu \nu \bar{A}_j^i \gamma_\mu^i \mathcal{A}_{i\ell} \gamma_\nu^j \mathcal{A}_\ell = \mu \nu \bar{A}_j^i \mathcal{A}_{i\ell} \gamma_\mu^i \mathcal{A}_\ell \gamma_\nu^j \mathcal{A}_\ell = \mu \nu \bar{A}_j^i \mathcal{A}_{i\ell} \gamma_\mu^i \mathcal{A}_\ell \gamma_\nu^j \mathcal{A}_\ell - \nu \mathcal{A}_\mu \mathcal{A}_\nu \gamma_\mu^j \gamma_\nu^i \mathcal{A}_\ell \\
&= \mu \nu \bar{A}_j^i \mathcal{A}_{i\ell} \underbrace{\gamma_\mu^i \mathcal{A}_\ell \gamma_\nu^j \mathcal{A}_\ell - \gamma_\nu^j \mathcal{A}_\ell \gamma_\mu^i \mathcal{A}_\ell}_{\gamma_\mu^i \mathcal{A}_\ell \gamma_\nu^j \mathcal{A}_\ell - \gamma_\nu^j \mathcal{A}_\ell \gamma_\mu^i \mathcal{A}_\ell} = \mu \nu \bar{A}_j^i \mathcal{A}_{i\ell} \gamma_\mu^i \mathcal{A}_\ell \gamma_\nu^j \mathcal{A}_\ell - \nu \mathcal{A}_\mu \mathcal{A}_\nu \gamma_\mu^j \gamma_\nu^i \mathcal{A}_\ell
\end{aligned}$$