

$$d\mathcal{A} = \bar{\mathcal{A}}$$

$$\left\{ \begin{array}{ll} \eta_{ij} = \mathcal{A}_i \mathcal{A}_j = \mathcal{A}_i \mathcal{A}_j & \bar{\eta}_{\ell}^m = \left( \bar{\mathcal{A}} \mathcal{A} \right)_{\ell}^m \\ \mathcal{A}_{\mu\nu} = \mathcal{A}_{\mu} \mathcal{A}_{\nu} = \mathcal{A}_{\mu} \mathcal{A}_{\nu} & \bar{\mathcal{A}}_{\ell}^m = \left( \bar{\mathcal{A}} \mathcal{A} \right)_{\ell}^m \\ m_n \eta = \mathcal{A} = \mathcal{A}_n \mathcal{A} = \mathcal{A}_n \mathcal{A} \text{ cst} & \bar{\mathcal{A}}_{\ell}^m = \left( \bar{\mathcal{A}} \mathcal{A} \right)_{\ell}^m \end{array} \right.$$

$$2 \underbrace{\bar{\eta}_{\ell}^m}_{nk} \mathcal{A} = \mathcal{A}_{\ell} | \bar{\mathcal{A}}_{mk} + \mathcal{A}_m | \bar{\mathcal{A}}_{\ell k} - \mathcal{A}_k | \bar{\mathcal{A}}_{\ell m} + \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{nk} \mathcal{A} - \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{n\ell} \mathcal{A} - \underbrace{\mathcal{A}_{\ell} \mathcal{A}_m | \bar{\mathcal{A}}^n}_{nm} \mathcal{A}$$

$$\underbrace{\bar{\eta}_{\ell}^m}_{nk} \mathcal{A} = \alpha \underbrace{\mathcal{A}_{\ell} | \bar{\mathcal{A}}_{mk}}_{= \mathcal{A}_{\ell} \times \mathcal{A}_{mk}} + \beta \underbrace{\mathcal{A}_m | \bar{\mathcal{A}}_{\ell k}}_{= \mathcal{A}_m \times \mathcal{A}_{\ell k}} + \gamma \underbrace{\mathcal{A}_k | \bar{\mathcal{A}}_{\ell m}}_{= \mathcal{A}_k \times \mathcal{A}_{\ell m}} + \lambda \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{nk} \mathcal{A} + \mu \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{n\ell} \mathcal{A} + \nu \underbrace{\mathcal{A}_{\ell} \mathcal{A}_m | \bar{\mathcal{A}}^n}_{nm} \mathcal{A}$$

$$\begin{aligned} 2 \underbrace{\bar{\mathcal{A}}_{\ell}^m}_{nk} \mathcal{A} &= \mathcal{A}_{\ell} | \bar{\mathcal{A}}_{mk} + \mathcal{A}_m | \bar{\mathcal{A}}_{\ell k} - \mathcal{A}_k | \bar{\mathcal{A}}_{\ell m} + \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{nk} \mathcal{A} - \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{n\ell} \mathcal{A} - \underbrace{\mathcal{A}_{\ell} \mathcal{A}_m | \bar{\mathcal{A}}^n}_{nm} \mathcal{A} \\ &= \mathcal{A}_{\ell} | \bar{\mathcal{A}}_{mk} + \mathcal{A}_m | \bar{\mathcal{A}}_{\ell k} - \mathcal{A}_k | \bar{\mathcal{A}}_{\ell m} \end{aligned}$$

$$\begin{aligned} 2 \underbrace{\bar{\mathcal{A}}_{\ell}^m}_{nk} \mathcal{A} &= \mathcal{A}_{\ell} | \bar{\mathcal{A}}_{mk} + \mathcal{A}_m | \bar{\mathcal{A}}_{\ell k} - \mathcal{A}_k | \bar{\mathcal{A}}_{\ell m} + \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{nk} \mathcal{A} - \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{n\ell} \mathcal{A} - \underbrace{\mathcal{A}_{\ell} \mathcal{A}_m | \bar{\mathcal{A}}^n}_{nm} \mathcal{A} \\ &= \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{nk} \mathcal{A} - \underbrace{\mathcal{A}_m \mathcal{A}_{\ell} | \bar{\mathcal{A}}^n}_{n\ell} \mathcal{A} - \underbrace{\mathcal{A}_{\ell} \mathcal{A}_m | \bar{\mathcal{A}}^n}_{nm} \mathcal{A} \end{aligned}$$

$$\text{metric} \begin{cases} \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} = \bar{\eta}_{ij} & \beta = 1/2: \gamma = -\alpha: \mu = -\lambda \\ \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} = \bar{\eta}_{ij} & \beta = 1/2: \gamma = -\alpha \\ \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} = \bar{\eta}_{ij} & \mu = -\lambda \end{cases}$$

$$\begin{aligned} \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} &= \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} = \alpha \bar{\eta}_{ij} + \beta \bar{\eta}_{ij} + \gamma \bar{\eta}_{ik} \\ + \lambda \underbrace{\bar{\eta}_k^m \eta_j}_{nj} + \mu \underbrace{\bar{\eta}_j^m \eta_k}_{ni} + \nu \underbrace{\bar{\eta}_j^m \eta_k}_{nk} + \alpha \bar{\eta}_{ki} + \beta \bar{\eta}_{ji} + \gamma \bar{\eta}_{jk} \\ + \lambda \underbrace{\bar{\eta}_k^m \eta_j}_{nj} + \mu \underbrace{\bar{\eta}_i^m \eta_k}_{ni} + \nu \underbrace{\bar{\eta}_j^m \eta_k}_{nk} &= \bar{\eta}_{ij} \\ \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} &= \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} = \\ \alpha \bar{\eta}_{ij} + \beta \bar{\eta}_{ij} + \gamma \bar{\eta}_{ik} + \alpha \bar{\eta}_{ki} + \beta \bar{\eta}_{ji} + \gamma \bar{\eta}_{jk} &= \bar{\eta}_{ij} \\ \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} &= \underbrace{\bar{\eta}_i^m \eta_j}_{mj} + \underbrace{\bar{\eta}_j^m \eta_i}_{mi} \\ = \lambda \underbrace{\bar{\eta}_k^m \eta_j}_{nj} + \mu \underbrace{\bar{\eta}_j^m \eta_k}_{ni} + \nu \underbrace{\bar{\eta}_j^m \eta_k}_{nk} & \\ + \lambda \underbrace{\bar{\eta}_k^m \eta_j}_{nj} + \mu \underbrace{\bar{\eta}_i^m \eta_k}_{ni} + \nu \underbrace{\bar{\eta}_j^m \eta_k}_{nk} &= \bar{\eta}_{ij} \end{aligned}$$

$$\text{tors free} \begin{cases} \bar{q}^i - q^j \sum_j \bar{q}^i \eta^i = 0 & \alpha = \beta: \lambda = 1/2: \mu = \nu \\ \bar{r}^j \sum_j \bar{s}^i \eta^i = 0 & \alpha = \beta \\ \bar{t}^i - t^j \sum_j \bar{s}^i \eta^i = 0 & \lambda = 1/2: \mu = \nu \end{cases}$$

$$\begin{aligned} & \overbrace{p \bar{t}_q \bar{t} | \bar{r}^j \sum_j \bar{q}^i \eta^i}_{il} = \overbrace{p \bar{t}_q \bar{r}^j}_{q\ell} \overbrace{\bar{t} | \bar{q}^i \eta^i}_{il} - \overbrace{q \bar{t}_q \bar{r}^j}_{p\ell} \overbrace{\bar{t} | \bar{q}^i \eta^i}_{il} = \overbrace{\bar{t} | \bar{q}^i \eta^i}_{p\ell} - \overbrace{\bar{t} | \bar{q}^i \eta^i}_{q\ell} \\ & = \bar{q}^i \bar{r}^j \eta^i \eta^j - \bar{p}^i \bar{q}^j \bar{r}^i \eta^i \eta^j = \alpha \bar{p} \bar{t} | \bar{q}^i + \beta \bar{q} \bar{t} | \bar{p}^i + \gamma \bar{r} \bar{t} | \bar{q}^i + \lambda \overbrace{p \bar{t}_q \bar{t} | \bar{r}^n}_{n\ell} + \mu \overbrace{\bar{t} | \bar{q}^i \eta^i}_{np} + \nu \overbrace{\bar{t} | \bar{q}^i \eta^i}_{nq} \\ & - \alpha \bar{q} \bar{t} | \bar{p}^i - \beta \bar{p} \bar{t} | \bar{q}^i - \gamma \bar{r} \bar{t} | \bar{q}^i - \lambda \overbrace{q \bar{t}_p \bar{t} | \bar{r}^n}_{n\ell} - \mu \overbrace{\bar{t} | \bar{q}^i \eta^i}_{nq} - \nu \overbrace{\bar{t} | \bar{q}^i \eta^i}_{np} = \overbrace{\bar{t} | \bar{q}^i \eta^i}_{n\ell} \\ & \overbrace{p \bar{t}_q \bar{t} | \bar{r}^j \sum_j \bar{q}^i \eta^i}_{il} = \overbrace{p \bar{t}_q \bar{r}^j}_{q\ell} \overbrace{\bar{t} | \bar{q}^i \eta^i}_{il} - \overbrace{q \bar{t}_q \bar{r}^j}_{p\ell} \overbrace{\bar{t} | \bar{q}^i \eta^i}_{il} = \overbrace{\bar{t} | \bar{q}^i \eta^i}_{p\ell} - \overbrace{\bar{t} | \bar{q}^i \eta^i}_{q\ell} \\ & = \bar{q}^i \bar{r}^j \eta^i \eta^j - \bar{p}^i \bar{q}^j \bar{r}^i \eta^i \eta^j = \alpha \bar{p} \bar{t} | \bar{q}^i + \beta \bar{q} \bar{t} | \bar{p}^i + \gamma \bar{r} \bar{t} | \bar{q}^i - \alpha \bar{q} \bar{t} | \bar{p}^i - \beta \bar{p} \bar{t} | \bar{q}^i - \gamma \bar{r} \bar{t} | \bar{q}^i = 0 \\ & \overbrace{p \bar{r}_q \bar{r} | \bar{t}^j \sum_j \bar{s}^i \eta^i}_{il} = \overbrace{p \bar{r}_q \bar{t}^j}_{q\ell} \overbrace{\bar{r} | \bar{s}^i \eta^i}_{il} - \overbrace{q \bar{r}_q \bar{t}^j}_{p\ell} \overbrace{\bar{r} | \bar{s}^i \eta^i}_{il} = \overbrace{\bar{r} | \bar{s}^i \eta^i}_{p\ell} - \overbrace{\bar{r} | \bar{s}^i \eta^i}_{q\ell} \\ & = \bar{q}^i \bar{r}^j \eta^i \eta^j - \bar{p}^i \bar{q}^j \bar{r}^i \eta^i \eta^j = \bar{q}^i \bar{r}^j \eta^i \eta^j - \bar{p}^i \bar{q}^j \bar{r}^i \eta^i \eta^j \\ & = \lambda \overbrace{p \bar{r}_q \bar{r} | \bar{t}^n}_{n\ell} + \mu \overbrace{\bar{r} | \bar{s}^i \eta^i}_{np} + \nu \overbrace{\bar{r} | \bar{s}^i \eta^i}_{nq} - \lambda \overbrace{q \bar{r}_p \bar{r} | \bar{t}^n}_{n\ell} - \mu \overbrace{\bar{r} | \bar{s}^i \eta^i}_{nq} - \nu \overbrace{\bar{r} | \bar{s}^i \eta^i}_{np} = \overbrace{\bar{r} | \bar{s}^i \eta^i}_{n\ell} \end{aligned}$$

$$\mathfrak{h} \xrightarrow{\mathbb{1}} \mathbb{C}_n \mathbb{R}^n$$

$$\begin{aligned} 0 &= d_m \delta^n = d \overbrace{m \bar{r}^\lambda}_{\lambda} \bar{t}^n = \overbrace{m \bar{r}^\lambda}_{\lambda} \bar{t}^n + \overbrace{m \bar{r}^\lambda}_{\lambda} \bar{t}^n \\ m \bar{t}^n - m \bar{r}^\mu \bar{r}^\nu \bar{t}^n &= \overbrace{m \bar{r}^\lambda}_{\lambda} \bar{t}^n = - \overbrace{m \bar{r}^\lambda}_{\lambda} \bar{t}^n \end{aligned}$$