$$\mathcal{Q} \underset{\text{alg}}{\sqsubset} K \underset{\text{trans}}{\sqsubset} K_a(x{:}y) \sqsubset K(x{:}y)$$

$$K = \frac{f \in K_a(x{:}y)}{f\mathbb{Q}\text{ alg}}$$

$$\begin{array}{ccc} H_b(u{:}v) & \xleftarrow[\asymp]{\sigma} & K_a(x{:}y) \\ \cup & & \cup \\ H & \xleftarrow[\asymp]{\sigma} & K \end{array}$$

$$\mathbb{Q} \ni \pm\frac{1+\cdots+1}{1+\cdot\cdot+1} \Longrightarrow \sigma|\mathbb{Q} = \imath$$

$$H \xleftarrow[\asymp]{\sigma} K$$

$$K = \mathbb{Q}\left(\alpha_1{:}\cdots{:}\alpha_m\right)$$

$$\prod_i \underbrace{\lambda-\alpha_i}_{} \in \mathbb{Q}[\lambda] \Leftrightarrow \sigma_i\left(\alpha_1{:}\cdots{:}\alpha_m\right) \in \mathbb{Q}$$

$$\Rightarrow \mathbb{Q} \ni \sigma_i\left(\alpha_1{:}\cdots{:}\alpha_m\right) = {}^\sigma\overbrace{\sigma_i\left(\alpha_1{:}\cdots{:}\alpha_m\right)}^{} = \sigma_i\left({}^\sigma\alpha_1{:}\cdots{:}{}^s\alpha_m\right)$$

$$\Rightarrow \prod_i \underbrace{\lambda - {}^\sigma\alpha_i}_{} \in \mathbb{Q}[\lambda] \Rightarrow {}^\sigma\alpha_1{:}\cdots{}^\sigma\alpha_m \in H \text{ alg} \Rightarrow {}^\sigma K \sqsubset H \Rightarrow {}^\sigma K = H$$

$$
\begin{array}{ccc}
K(u{:}v) & \xleftarrow[\asymp]{\tau} & H(u{:}v) \\
\cup & & \cup \\
K_{\not\propto b}(u{:}v) & \xleftarrow[\asymp]{\tau} & H_b(u{:}v) \\
\cup & & \cup \\
K & \xleftarrow[\asymp]{\overset{-1}{\sigma}} & H
\end{array}
$$

$$
\tau \frac{a_{i:j}u^i v^j}{b_{m:n}u^m v^n} = \frac{{}^{\overset{-1}{\sigma}}a_{i:j}u^i v^j}{{}^{\overset{-1}{\sigma}}b_{m:n}u^m v^n}
$$

$$
\begin{array}{ccccc}
K(u{:}v) & \xleftarrow[\asymp]{\tau} & H(u{:}v) & & \\
\cup & & \cup & & \\
K_{\not\propto b}(u{:}v) & \xleftarrow[\asymp]{\tau} & H_b(u{:}v) & \xleftarrow[\asymp]{\sigma} & K_a(x{:}y) \\
\cup & & \cup & & \cup \\
K & \xleftarrow[\asymp]{\overset{-1}{\sigma}} & H & \xleftarrow[\asymp]{\sigma} & K
\end{array}
$$

$$
\mathcal{J}_K(a) = \mathcal{J}_K\left({}^{\not\propto}b\right)
$$

$$
\begin{array}{c|c} \alpha & \beta \\ \hline \gamma & \delta \end{array}
\begin{bmatrix} ገ \\ ቅ \end{bmatrix}_f = \begin{bmatrix} \overset{\alpha}{ገ} + \overset{\beta}{ቅ} \\ \overset{\gamma}{ገ} + \overset{\delta}{ቅ} \end{bmatrix}_g
$$

$$
{}^x f = \underbrace{1 - a^2x^2}\underbrace{a^2 - x^2}: \quad {}^u g = \underbrace{1 - b^2u^2}\underbrace{b^2 - u^2}
$$

$$\overset{\alpha}{\overbrace{\text{ገ1}}} = \overset{\alpha}{\text{ገ}}\overset{\alpha}{\text{1}} + g\overset{\gamma}{\text{ገ}}\overset{\gamma}{\text{1}}: \quad \overset{\alpha}{\overbrace{f\text{ት4}}} = \overset{\beta}{\text{ት}}\overset{\beta}{\text{4}} + g\overset{\delta}{\text{ት}}\overset{\delta}{\text{4}}$$

$$\overset{\beta}{\overbrace{\text{ገ4}}} = \overset{\alpha}{\text{ገ}}\overset{\beta}{\text{4}} + g\overset{\gamma}{\text{ገ}}\overset{\delta}{\text{4}}: \quad \overset{\beta}{\overbrace{\text{ት1}}} = \overset{\beta}{\text{ት}}\overset{\alpha}{\text{1}} + g\overset{\delta}{\text{ት}}\overset{\gamma}{\text{1}}$$

$$\overset{\gamma}{\overbrace{\text{ገ1}}} = \overset{\alpha}{\text{ገ}}\overset{\gamma}{\text{1}} + \overset{\gamma}{\text{ገ}}\overset{\alpha}{\text{1}}: \quad \overset{\gamma}{\overbrace{f\text{ት4}}} = \overset{\beta}{\text{ት}}\overset{\delta}{\text{4}} + \overset{\delta}{\text{ት}}\overset{\beta}{\text{4}}$$

$$\overset{\delta}{\overbrace{\text{ገ4}}} = \overset{\alpha}{\text{ገ}}\overset{\delta}{\text{4}} + \overset{\gamma}{\text{ገ}}\overset{\beta}{\text{4}}: \quad \overset{\delta}{\overbrace{\text{ት1}}} = \overset{\beta}{\text{ት}}\overset{\gamma}{\text{1}} + \overset{\delta}{\text{ት}}\overset{\alpha}{\text{1}}$$

$$\begin{bmatrix} \overset{\alpha}{\overbrace{\text{ገ1}}} + \overset{\alpha}{\overbrace{f\text{ት4}}} + \overset{\beta}{\overbrace{\text{ገ4}}} + \overset{\beta}{\overbrace{\text{ት1}}} \\ \overset{\gamma}{\overbrace{\text{ገ1}}} + \overset{\gamma}{\overbrace{f\text{ት4}}} + \overset{\delta}{\overbrace{\text{ገ4}}} + \overset{\delta}{\overbrace{\text{ት1}}} \end{bmatrix}_g = \frac{\alpha \mid \beta}{\gamma \mid \delta} \begin{bmatrix} \text{ገ1} + f\text{ት4} \\ \text{ገ4} + \text{ት1} \end{bmatrix}_f = \frac{\alpha \mid \beta}{\gamma \mid \delta} \overbrace{\begin{bmatrix} \text{ገ} \\ \text{ት} \end{bmatrix}_f \begin{bmatrix} \text{1} \\ \text{4} \end{bmatrix}_f} = \overbrace{\frac{\alpha \mid \beta}{\gamma \mid \delta} \begin{bmatrix} \text{ገ} \\ \text{ት} \end{bmatrix}_f} \overbrace{\frac{\alpha \mid \beta}{\gamma \mid \delta} \begin{bmatrix} \text{1} \\ \text{4} \end{bmatrix}_f}$$

$$= \begin{bmatrix} \overset{\alpha}{\text{ገ}} + \overset{\beta}{\text{ት}} \\ \overset{\gamma}{\text{ገ}} + \overset{\delta}{\text{ት}} \end{bmatrix}_g \begin{bmatrix} \overset{\alpha}{\text{1}} + \overset{\beta}{\text{4}} \\ \overset{\gamma}{\text{1}} + \overset{\delta}{\text{4}} \end{bmatrix}_g = \begin{bmatrix} \underbrace{\overset{\alpha}{\text{ገ}} + \overset{\beta}{\text{ት}}}\underbrace{\overset{\alpha}{\text{1}} + \overset{\beta}{\text{4}}} + g\underbrace{\overset{\gamma}{\text{ገ}} + \overset{\delta}{\text{ት}}}\underbrace{\overset{\gamma}{\text{1}} + \overset{\delta}{\text{4}}} \\ \underbrace{\overset{\alpha}{\text{ገ}} + \overset{\beta}{\text{ት}}}\underbrace{\overset{\gamma}{\text{1}} + \overset{\delta}{\text{4}}} + \underbrace{\overset{\gamma}{\text{ገ}} + \overset{\delta}{\text{ት}}}\underbrace{\overset{\alpha}{\text{1}} + \overset{\beta}{\text{4}}} \end{bmatrix}_g = \begin{bmatrix} \overset{\alpha}{\text{ገ}}\overset{\alpha}{\text{1}} + \overset{\alpha}{\text{ገ}}\overset{\beta}{\text{4}} + \overset{\beta}{\text{ት}}\overset{\alpha}{\text{1}} + \overset{\beta}{\text{ት}}\overset{\beta}{\text{4}} + g\overset{\gamma}{\text{ገ}}\overset{\gamma}{\text{1}} + g\overset{\gamma}{\text{ገ}}\overset{\delta}{\text{4}} + g\overset{\delta}{\text{ት}}\overset{\gamma}{\text{1}} + g\overset{\delta}{\text{ት}}\overset{\delta}{\text{4}} \\ \overset{\alpha}{\text{ገ}}\overset{\gamma}{\text{1}} + \overset{\alpha}{\text{ገ}}\overset{\delta}{\text{4}} + \overset{\beta}{\text{ት}}\overset{\gamma}{\text{1}} + \overset{\beta}{\text{ት}}\overset{\delta}{\text{4}} + \overset{\gamma}{\text{ገ}}\overset{\alpha}{\text{1}} + \overset{\gamma}{\text{ገ}}\overset{\beta}{\text{4}} + \overset{\delta}{\text{ት}}\overset{\alpha}{\text{1}} + \overset{\delta}{\text{ት}}\overset{\beta}{\text{4}} \end{bmatrix}_g$$

$$\overset{\alpha}{1} = 1: \quad \overset{\gamma}{1} = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}_g = \frac{\alpha \mid \beta}{\gamma \mid \delta} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_f = \begin{bmatrix} \overset{\alpha}{1} \\ \overset{\gamma}{1} \end{bmatrix}_g$$

$$\begin{cases} {}^{0}\overset{\alpha}{\text{ገ}} = {}^{0}\text{ገ} & {}^{0}\overset{\beta}{\text{ት}} = 0 \\ {}^{0}\overset{\gamma}{\text{ገ}} = 0 & b\,{}^{0}\overset{\delta}{\text{ት}} = a\,{}^{0}\text{ት} \end{cases}$$

$${}^{0}\text{ገ} \pm a\,{}^{0}\text{ት} = {}^{0:\pm a}\begin{bmatrix} \text{ገ} \\ \text{ት} \end{bmatrix}_f = {}^{0:\pm b}\overbrace{\frac{\alpha \mid \beta}{\gamma \mid \delta} \begin{bmatrix} \text{ገ} \\ \text{ት} \end{bmatrix}_f} = {}^{0:\pm b}\begin{bmatrix} \overset{\alpha}{\text{ገ}} + \overset{\beta}{\text{ት}} \\ \overset{\gamma}{\text{ገ}} + \overset{\delta}{\text{ት}} \end{bmatrix}_g = {}^{0}\overset{\alpha}{\text{ገ}} + {}^{0}\overset{\beta}{\text{ት}} \pm b\underbrace{{}^{0}\overset{\gamma}{\text{ገ}} + {}^{0}\overset{\delta}{\text{ት}}} \Rightarrow \begin{cases} {}^{0}\text{ገ} = {}^{0}\overset{\alpha}{\text{ገ}} + {}^{0}\overset{\beta}{\text{ት}} \\ a\,{}^{0}\text{ት} = b\underbrace{{}^{0}\overset{\gamma}{\text{ገ}} + {}^{0}\overset{\delta}{\text{ት}}} \end{cases}$$

$${}^{0}\overset{\alpha}{f} = a^2$$

$${}^{0}f = a^2: \quad {}^{0}g = b^2: \quad {}^{\pm\frac{\pm}{a}}f = 0: \quad {}^{\pm\frac{\pm}{b}}g = 0$$

$${}^{x:y}\begin{bmatrix} \text{ገ} \\ \text{ት} \end{bmatrix}_f = {}^{x}\text{ገ} + y\underbrace{1 - a^2x^2}\,{}^{x}\text{ት} = {}^{x}\text{ገ} + \frac{a^2 - x^2}{y}\,{}^{x}\text{ት}$$

$$ {}^{u:v}\begin{bmatrix}\mathsf{1}\\ \mathsf{4}\end{bmatrix}_g = {}^{u}\mathsf{1} + v\underbrace{1 - \overset{2}{b}u^2}\,{}^{u}\mathsf{4} = {}^{u}\mathsf{1} + \frac{\overset{2}{b} - u^2}{v}\,{}^{u}\mathsf{4} $$

$$ {}^{\pm\overset{\pm}{a}}\text{ɿ} = {}^{\pm\overset{\pm}{b}}\overset{\alpha}{\text{ɿ}} : \quad 0 = {}^{\pm\overset{\pm}{b}}\overset{\beta}{\text{ƭ}} $$

$$ {}^{\pm a:0}\begin{bmatrix}\text{ɿ}\\ \text{ƭ}\end{bmatrix}_f = {}^{\pm a}\text{ɿ} : \quad {}^{\pm\overset{-1}{a}:\infty}\begin{bmatrix}\text{ɿ}\\ \text{ƭ}\end{bmatrix}_f = {}^{\pm\overset{-1}{a}}\text{ɿ} : \quad {}^{\pm b:0}\begin{bmatrix}\mathsf{1}\\ \mathsf{4}\end{bmatrix}_g = {}^{\pm b}\mathsf{1} : \quad {}^{\pm\overset{-1}{b}:\infty}\begin{bmatrix}\mathsf{1}\\ \mathsf{4}\end{bmatrix}_g = {}^{\pm\overset{-1}{b}}\mathsf{1} $$

$$ {}^{\pm a}\text{ɿ} = {}^{\pm a:0}\begin{bmatrix}\text{ɿ}\\ \text{ƭ}\end{bmatrix}_f = {}^{\pm b:0}\overbrace{\begin{array}{c|c}\alpha & \beta\\ \hline \gamma & \delta\end{array}\begin{bmatrix}\text{ɿ}\\ \text{ƭ}\end{bmatrix}_f} = {}^{\pm b:0}\begin{bmatrix}\overset{\alpha}{\text{ɿ}} + \overset{\beta}{\text{ƭ}}\\ \overset{\gamma}{\text{ɿ}} + \overset{\delta}{\text{ƭ}}\end{bmatrix}_g = {}^{\pm b}\overset{\alpha}{\text{ɿ}} + {}^{\pm b}\overset{\beta}{\text{ƭ}} $$

$$ {}^{\pm\overset{-1}{a}}\text{ɿ} = {}^{\pm\overset{-1}{a}:\infty}\begin{bmatrix}\text{ɿ}\\ \text{ƭ}\end{bmatrix}_f = {}^{\pm\overset{-1}{b}:\infty}\overbrace{\begin{array}{c|c}\alpha & \beta\\ \hline \gamma & \delta\end{array}\begin{bmatrix}\text{ɿ}\\ \text{ƭ}\end{bmatrix}_f} = {}^{\pm\overset{-1}{b}:\infty}\begin{bmatrix}\overset{\alpha}{\text{ɿ}} + \overset{\beta}{\text{ƭ}}\\ \overset{\gamma}{\text{ɿ}} + \overset{\delta}{\text{ƭ}}\end{bmatrix}_g = {}^{\pm\overset{-1}{b}}\overset{\alpha}{\text{ɿ}} + {}^{\pm\overset{-1}{b}}\overset{\beta}{\text{ƭ}} $$

$$ \overset{\alpha}{f} = \overset{\beta}{1}{}^2 + g\,\overset{\delta}{1}{}^2 $$

$$ \begin{bmatrix}\overset{\alpha}{f}\\ \overset{\gamma}{f}\end{bmatrix}_g = \begin{array}{c|c}\alpha & \beta\\ \hline \gamma & \delta\end{array}\begin{bmatrix}f\\ 0\end{bmatrix}_f = \begin{array}{c|c}\alpha & \beta\\ \hline \gamma & \delta\end{array}\overbrace{\begin{bmatrix}0\\ 1\end{bmatrix}_f\begin{bmatrix}0\\ 1\end{bmatrix}_f} = \overbrace{\begin{array}{c|c}\alpha & \beta\\ \hline \gamma & \delta\end{array}\begin{bmatrix}0\\ 1\end{bmatrix}_f}\overbrace{\begin{array}{c|c}\alpha & \beta\\ \hline \gamma & \delta\end{array}\begin{bmatrix}0\\ 1\end{bmatrix}_f} = \begin{bmatrix}\overset{\beta}{1}\\ \overset{\delta}{1}\end{bmatrix}_g\begin{bmatrix}\overset{\beta}{1}\\ \overset{\delta}{1}\end{bmatrix}_g = \begin{bmatrix}\overset{\beta}{1}{}^2 + g\overset{\delta}{1}{}^2\\ 2\overset{\beta}{1}\overset{\delta}{1}\end{bmatrix}_g $$

$$ {}^{\pm\overset{\pm}{b}}\overset{\alpha}{f} = 0 $$

$$ {}^{\pm\overset{\pm}{b}}\overset{\alpha}{f} = \underbrace{{}^{\pm\overset{\pm}{b}}\overset{\beta}{1}{}^2}_{=0} + \underbrace{{}^{\pm\overset{\pm}{b}}g}_{=0}\,{}^{\pm\overset{\pm}{b}}\overset{\delta}{1}{}^2 $$