$$\begin{array}{c|c} a & b \\ \hline c & d \end{array} \in {}^{2}\mathbb{R}_{2}^{\mathsf{C}}$$

$$1|\tau \begin{array}{c|c} a & b \\ \hline c & d \end{array} = a + \tau c | b + \tau d = \frac{b + \tau d}{a + \tau c}$$

$$\begin{array}{c|c} \alpha & \beta \\ \hline \gamma & \delta \end{array} \in {}^{2}\mathbb{Z}_{2}^{\mathsf{C}}$$

$$\omega_1 | \omega_2 \begin{array}{c|c} \alpha & \beta \\ \hline \gamma & \delta \end{array} = \omega_1 \alpha + \omega_2 \gamma | \omega_1 \beta + \omega_2 \delta = \frac{\omega_1 \beta + \omega_2 \delta}{\omega_1 \alpha + \omega_2 \gamma} = \frac{\beta + \frac{\omega_2}{\omega_1}\delta}{\alpha + \frac{\omega_2}{\omega_1}\gamma} = 1 | \frac{\omega_2}{\omega_1} \begin{array}{c|c} \alpha & \beta \\ \hline \gamma & \delta \end{array}$$

$${}_{n}^{z}\Omega = z^{-2n} + \sum_{\omega}^{\Omega} \underbrace{\overbrace{z - \omega}^{-2n} - \omega^{-2n}}$$

$${}_{n}^{\infty}\Omega = -\sum_{\omega}^{\Omega} \frac{1}{\omega^{2n}}$$

$${}_{1}^{z}\Omega = z^{-2} + \sum_{\omega}^{\Omega \llcorner 0} \underbrace{\overbrace{z - \omega}^{-2} - \omega^{-2}}$$

$${}^{\mathbb{C}}{\triangle}_{m}\mathbb{C} \ni {}_{1}\Omega \in {}^{\mathbb{C} \ulcorner \Omega}{\triangle}_{\omega}\mathbb{C}$$

$${}_{1}\Omega \in {}^{\mathbb{C} \ulcorner \Omega}{\triangle}_{m}\mathbb{C} \ni {}_{1}\underline{\Omega}$$

$$
{}^{z}_{1}\Omega \,=\, z^{-2} - \sum_{m \geqslant 1} (2m+1)\, z^{2m}\; {}^{\infty}_{m+1}\Omega
$$

$$
{}^{\tau}_{z}\wp \,=\, \frac{1}{z^2} + \sum_{k \geqslant 1} {}^{\tau}_{\sharp}\wp^{2k}\, \frac{1}{z^{2k}}
$$

$$
\overbrace{z-\omega}^{-2} - \omega^{-2} \,=\, \omega^{-2} \left(\overbrace{\frac{z}{\omega} - 1}^{-2} - 1 \right) \,=\, \omega^{-2} \sum_{n \geqslant 1} (n+1)\, \frac{z^n}{\omega^n} = \sum_{n \geqslant 1} (n+1)\, \frac{z^n}{\omega^{n+2}}
$$

$$
\Longrightarrow {}^{z}\wp \,=\, z^{-2} + \sum_{\omega}^{\Lambda \llcorner 0} \left(\overbrace{z-\omega}^{-2} - \omega^{-2} \right) \,=\, z^{-2} + \sum_{\omega}^{\Lambda \llcorner 0} \sum_{n \geqslant 1} (n+1)\, \frac{z^n}{\omega^{n+2}}
$$

$$
=\, z^{-2} + \sum_{n \geqslant 1} (n+1)\, z^n \sum_{\omega}^{\Lambda \llcorner 0} \omega^{-n-2} \,=\, z^{-2} + \sum_{m \geqslant 1} (2m+1)\, z^{2m} \sum_{\omega}^{\Lambda \llcorner 0} \omega^{-2\,(m+1)}
$$

$$
\Longrightarrow {}_{\sharp}\wp^{2k} = (2k+1) \sum_{\omega}^{\Lambda \llcorner 0} \omega^{-2k-2}
$$

$$
{}^{\tau}_{\sharp}\wp^{2k} = (2k+1)\, G_{k+1}
$$

$$
{}^{\tau}_{\sharp}\wp^{2} = 3 \sum_{\omega}^{\Lambda \llcorner 0} \omega^{-4}
$$

$$
{}^{\tau}_{\sharp}\wp^{4} = 5 \sum_{\omega}^{\Lambda \llcorner 0} \omega^{-6}
$$

$$
y^2 = 4\,x^3 - 60\,G_4\,x - 140\,G_6 = 4\,x^3 - 20\,{}_{\sharp}\wp^2\,x - 28\,{}_{\sharp}\wp^4
$$

$$
(y/2)^2 \,=\, x^3 - 5\,{}_{\sharp}\wp^2\,x - 7\,{}_{\sharp}\wp^4
$$

$$\overbrace{a+\tau c}^{-1}\overbrace{b+\tau d}\underbrace{\wp}^{k} = \overbrace{a+\tau c}^{k/2}\ {}^{\tau}\underbrace{\wp}^{k}$$

$$(2k \overset{-1}{+} 1)\ \text{LHS} = \sum_{m:n}^{{}_2\mathbb{Z}\llcorner 0} \overbrace{m + \frac{b+\tau d}{a+\tau c} n}^{-k} = \overbrace{a+\tau c}^{k} \sum_{m:n}^{{}_2\mathbb{Z}\llcorner 0} \overbrace{\underbrace{a+\tau c} m + \underbrace{b+\tau d} n}^{-k}$$

$$= \overbrace{a+\tau c}^{k} \sum_{m:n}^{{}_2\mathbb{Z}\llcorner 0} \overbrace{\underbrace{am+bn}_{=\acute{m}} + \tau \underbrace{cm+dn}_{=\acute{n}}}^{-k} = \overbrace{a+\tau c}^{k} \sum_{\acute{m}:\acute{n}}^{{}_2\mathbb{Z}\llcorner 0} \overbrace{\acute{m}+\tau\acute{n}}^{-k} = (2k \overset{-1}{+} 1)\ \text{RHS}$$

$$\frac{\acute{m}}{\acute{n}} = \frac{a \mid b}{c \mid d}\ \frac{m}{n}$$

$$\frac{m}{n} = \frac{d \mid -b}{-c \mid a}\ \frac{\acute{m}}{\acute{n}}$$

$$\overset{z}{\underset{1}{\underline{\Omega}}}{}^{2} = 4\,\overset{z}{\underset{1}{\Omega}}{}^{3} + 60\,\overset{\infty}{\underset{2}{\Omega}}\,\overset{z}{\underset{1}{\Omega}} + 140\,\overset{\infty}{\underset{3}{\Omega}}$$

$$z+\Omega \in \mathbb{C} \ulcorner \Omega \xrightarrow[\text{inj}]{\wp:\underline{\wp}:1} \mathbb{P}^2\mathbb{C} \ni \begin{cases} {}^{z}\wp:{}^{z}\underline{\wp}:1 & z \notin \Omega \\ 0:1:0 & z \in \Omega \end{cases}$$

$$\sum_{\omega}^{\Omega \llcorner 0} \ulcorner \omega \urcorner^{-\alpha} < \infty \Leftrightarrow \alpha > 2$$

$$\ulcorner \omega_1 \urcorner \overset{\alpha}{\curlywedge} \ulcorner \omega_2 \urcorner \sum_{\omega}^{\Omega \llcorner 0} \ulcorner \omega \urcorner^{-\alpha} \leqslant 8 \sum_{n \geqslant 1} n^{1-\alpha} \leqslant \overset{\alpha}{\ulcorner \omega_1 + \omega_2 \urcorner} \sum_{\omega}^{\Omega \llcorner 0} \ulcorner \omega \urcorner^{-\alpha}$$

$$L = -\omega_1 - \omega_2 || \omega_1 + \omega_2$$

$$\omega \in \Omega \cap Ln \Longrightarrow \underbrace{\ulcorner \omega_1 \urcorner \curlywedge \ulcorner \omega_2 \urcorner} n \leqslant \ulcorner \omega \urcorner \leqslant \ulcorner \omega_1 + \omega_2 \urcorner n \Longrightarrow \frac{\ulcorner \omega_1 \urcorner \curlywedge \ulcorner \omega_2 \urcorner}{\ulcorner \omega \urcorner} \leqslant \frac{1}{n} \leqslant \frac{\ulcorner \omega_1 + \omega_2 \urcorner}{\ulcorner \omega \urcorner}$$

$$\Longrightarrow \ulcorner \omega_1 \urcorner \overset{\alpha}{\curlywedge} \ulcorner \omega_2 \urcorner \ulcorner \omega \urcorner^{-\alpha} \leqslant n^{-\alpha} \leqslant \overset{\alpha}{\ulcorner \omega_1 + \omega_2 \urcorner} \ulcorner \omega \urcorner^{-\alpha}$$

$$|\Omega \cap Ln| = 8n \Longrightarrow \ulcorner \omega_1 \urcorner \overset{\alpha}{\curlywedge} \ulcorner \omega_2 \urcorner \sum_{\omega}^{\Omega \cap Ln} \ulcorner \omega \urcorner^{-\alpha} \leqslant 8 n^{1-\alpha} \leqslant \overset{\alpha}{\ulcorner \omega_1 + \omega_2 \urcorner} \sum_{\omega}^{\Omega \cap Ln} \ulcorner \omega \urcorner^{-\alpha}$$

$$\Omega = \bigcup_{n \geqslant 0} \Omega \cap nL \Longrightarrow \ulcorner \omega_1 \urcorner \overset{\alpha}{\curlywedge} \ulcorner \omega_2 \urcorner \sum_{\omega}^{\Omega \llcorner 0} \ulcorner \omega \urcorner^{-\alpha} = \ulcorner \omega_1 \urcorner \overset{\alpha}{\curlywedge} \ulcorner \omega_2 \urcorner \sum_{n \geqslant 1} \sum_{\omega}^{\Omega \cap Ln} \ulcorner \omega \urcorner^{-\alpha} = \sum_{n \geqslant 1} \ulcorner \omega_1 \urcorner \overset{\alpha}{\curlywedge} \ulcorner \omega_2 \urcorner \sum_{\omega}^{\Omega \cap Ln} \ulcorner \omega \urcorner^{-\alpha}$$

$$\leqslant 8 \sum_{n \geqslant 1} n^{1-\alpha} \leqslant \sum_{n \geqslant 1} \overset{\alpha}{\ulcorner \omega_1 + \omega_2 \urcorner} \sum_{\omega}^{\Omega \cap Ln} \ulcorner \omega \urcorner^{-\alpha} = \overset{\alpha}{\ulcorner \omega_1 + \omega_2 \urcorner} \sum_{n \geqslant 1} \sum_{\omega}^{\Omega \cap Ln} \ulcorner \omega \urcorner^{-\alpha} = \overset{\alpha}{\ulcorner \omega_1 + \omega_2 \urcorner} \sum_{\omega}^{\Omega \llcorner 0} \ulcorner \omega \urcorner^{-\alpha}$$