$$y^2 \underbrace{1 - a^2 x^2} = a^2 - x^2$$

$$u^2 + v^2 = a^2 + a^2\, u^2\, v^2$$

$$\overbrace{y - v}^{2} + 2v\overbrace{y - v} + \underbrace{1 - a^2 v^2}\frac{x^2 - u^2}{1 - a^2 x^2} = 0$$

$$\overbrace{\underbrace{y - v} + v}^{2}\,\underbrace{1 - a^2 x^2} = a^2 - x^2$$

$$\underbrace{\overbrace{y - v}^{2} + v^2 + 2v\overbrace{y - v}}\,\underbrace{1 - a^2 x^2} = a^2 - x^2$$

$$\overbrace{y - v}^{2}\,\underbrace{1 - a^2 x^2} + v^2\,\underbrace{1 - a^2 x^2} + 2v\overbrace{y - v}\,\underbrace{1 - a^2 x^2} = a^2 - x^2$$

$$\overbrace{y - v}^{2}\,\underbrace{1 - a^2 x^2} + 2v\overbrace{y - v}\,\underbrace{1 - a^2 x^2} = a^2 - v^2 - x^2\,\underbrace{1 - a^2 v^2} = u^2\,\underbrace{1 - a^2 v^2} - x^2\,\underbrace{1 - a^2 v^2} = \underbrace{u^2 - x^2}\,\underbrace{1 - a^2 v^2}$$

$$y - v = \sqrt{v^2 - \underbrace{1 - a^2 v^2}\frac{x^2 - u^2}{1 - a^2 x^2}} - v = v\left(\sqrt{1 + \frac{a^2 v^2 - 1}{v^2}\frac{x^2 - u^2}{1 - a^2 x^2}} - 1\right)$$

$$= \sum_{n\,\geqslant\,1}\begin{bmatrix} 1/2 \\ n \end{bmatrix}\frac{\left(a^2 v^2 - 1\right)^n}{v^{2n\,-\,1}}\frac{\left(x^2 - u^2\right)^n}{\left(1 - a^2 x^2\right)^n} = \sum_{n\,\geqslant\,1}\begin{bmatrix} 1/2 \\ n \end{bmatrix}\frac{\left(a^2 v^2 - 1\right)^n}{v^{2n\,-\,1}}\frac{\left(\overbrace{x - u}^{2} + 2u\overbrace{x - u}\right)^n}{\left(1 - a^2 u^2 - a^2\overbrace{x - u}^{2} - 2a^2 u\overbrace{x - u}\right)^n}$$