

$${}^u\mathbb{Z} = \sum_n \mathcal{E}^{-\pi u n^2} = \sum_n \mathcal{E}^{-\pi n^2 u}$$

$${}^{u^{-1}}\mathbb{Z} = u^{1/2} {}^u\mathbb{Z}$$

$${}^z\Theta = \sum_n \mathcal{E}^{2\pi i z n^2} = \sum_n \mathcal{E}^{2\pi i n^2 z}$$

$$\Theta \in \mathcal{I}z > 0 \quad \triangleleft_{\omega} \mathbb{C}$$

$${}^{-1/4z}\Theta = \overline{2z/i}^{1/2} {}^z\Theta$$

$$\mathcal{I}\tau > 0$$

$${}^{\tau}\Theta = \sum_n \mathcal{E}^{\pi i \tau n^2}$$

$$\frac{1}{2} \text{ auto } \frac{1+2\mathbb{Z}}{2\mathbb{Z}} \Big| \frac{2\mathbb{Z}}{1+2\mathbb{Z}}$$

$${}^{-\tau^{-1}}\Theta = (-i\tau)^{1/2} {}^{\tau}\Theta$$

$${}^u\Theta_4 = \prod_{n \geq 1} (1 - 2^{2u} q^{2n-1} + q^{4n-2}) \prod_{n \geq 1} (1 - q^{2n})$$

$$\frac{{}^u\Theta_4}{{}^u\Theta_4} = 4 \sum_{n \geq 1} \frac{q^n}{1 - q^{2n}} {}^{2nu}\mathfrak{S}$$

$$\frac{{}^u\Theta_1}{{}^u\Theta_1} = {}^u\mathfrak{t}^{-1} + 4 \sum_{n \geq 1} \frac{q^{2n}}{1 - q^{2n}} {}^{2nu}\mathfrak{S}$$

$$\frac{{}^u\Theta_3}{{}^u\Theta_3} = 4 \sum_{n \geq 1} \frac{(-1)^n q^n}{1 - q^{2n}} {}^{2nu}\mathfrak{S}$$

$$\frac{{}^u\Theta_2}{{}^u\Theta_2} = -{}^u\mathfrak{t} + 4 \sum_{n \geq 1} \frac{(-1)^n q^{2n}}{1 - q^{2n}} {}^{2nu}\mathfrak{S}$$

$$\prod_{k \geq 1} (1 - tx^k) = \sum_n \frac{(-1)^n x^{n(n+1)/2} t^n}{\prod_{1 \leq k \leq n} (1 - x^k)} = \sum_n \prod_{1 \leq k \leq n} \frac{(-t) x^k}{1 - x^k}$$

$$\prod_{k \geq 1} \frac{1}{1 - tx^k} = \sum_n \frac{t^n}{\prod_{1 \leq k \leq n} (1 - x^k)} = \sum_n \prod_{1 \leq k \leq n} \frac{t}{1 - x^k}$$

$$1 - 2q + 2q^4 - 2q^9 + \dots = {}^0\Theta_4 = \prod_{n \geq 1} \overbrace{1 - q^{2n-1}}^2 \prod_{n \geq 1} (1 - q^{2n})$$

$$1 - 2q^4 + 2q^{16} - \dots = \prod_{n \geq 1} (1 + q^{4n-2}) \prod_{n \geq 1} (1 - q^{2n})$$