

$$\mathbb{h}_{\infty}^{\mathbb{h}} \mathbb{h}_{\infty}^{\mathbb{h}} \mathbb{K}^{\mathbb{N}} \xleftarrow{\mathbb{L}'} \underbrace{\mathbb{h}_{\infty}^{\mathbb{h}} \mathbb{K}}_{2^n}$$

$$\mathbb{h}_{\infty}^{\mathbb{h}} \mathbb{h}_{\infty}^{\mathbb{h}} \mathbb{K}^{\mathbb{N}} \ni \mathbb{L}^J = \sum_{j \in J} \mathbb{L}^j \quad \text{dual standard basis}$$

$$\mathbb{L}^I \mathbb{L}^J = \det \mathbb{L}^i \mathbb{L}^j = \det {}_i \delta^j = {}_I \delta^J = {}_I \mathbb{L} \mathbb{L}^J$$

$$\mathbb{L}^I = {}_I \mathbb{L}$$

$$\mathbb{L}^I \times \mathbb{L}^J = \mathbb{L}^I \overset{\circ}{\eta} \mathbb{L}^J = \det \mathbb{L}^i \times \mathbb{L}^j = \det {}_i \eta^j = {}_I \eta^J = {}_I \overset{\circ}{\eta}^J$$

$$\times {}_I \mathbb{L} = \mathbb{L}^I {}_I \eta^I$$

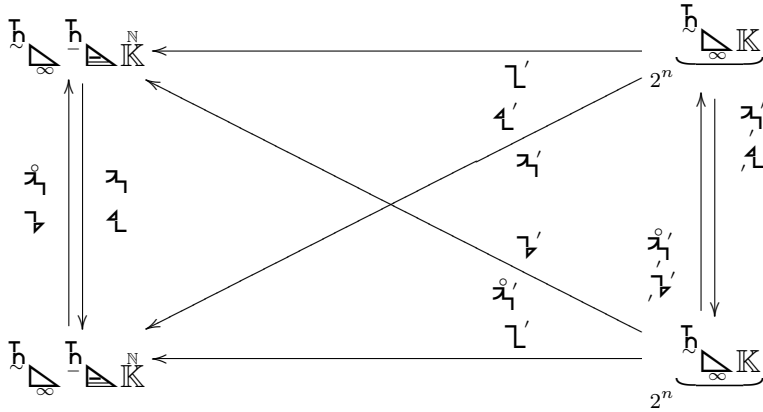
$$\mathbb{L}^I = (\times {}_I \mathbb{L}) {}_I \eta^I$$

$$\ast \mathbb{L}^I = \mathbb{L}^{N-I} \overset{I > \overline{N-I}}{\circ} {}_I \eta^I$$

$$\mathbb{L}^I = \mathbb{L} \underbrace{\mathbb{L}^I \mathbb{L}^J}_{\mathbb{L}^I \mathbb{L}^J}: \quad {}_M \mathbb{L} \mathbb{L}^N = \det ({}_{\mu} \mathbb{L} \mathbb{L}^{\nu}) = \det {}_{\mu} \delta^{\nu} = {}_M \delta^N$$

$$\mathbb{L}^I = \mathbb{L} \underbrace{\mathbb{L}^I \mathbb{L}^J}_{\mathbb{L}^I \mathbb{L}^J}: \quad \mathbb{L}^I \mathbb{L}^J = \det \mathbb{L}^i \mathbb{L}^j = \det {}_i \delta^j = {}_I \delta^J = {}_I \mathbb{L} \mathbb{L}^J$$

$$\mathbb{L}^I = {}_I \mathbb{L}$$



$$\mathbb{L}^I \times \mathbb{L}^J = \begin{cases} \mathbb{L}^I \mathbb{L}^J = \mathbb{L}^{IJ} \\ \mathbb{L}^I \mathbb{L}^J = {}_I \mathbb{L} \mathbb{L}^J = \det \mathbb{L}^i \times \mathbb{L}^j = \det {}_i \eta^j = \det {}_I \eta^J = {}_I \mathbb{L}^J \end{cases}$$

$$\times {}_J \mathbb{L} = \sum_{|I|=|J|} \mathbb{L}^I \times {}_I \mathbb{L}^J$$

$$\mathbb{1}^J = \sum_{|I|=|J|} \left(\overset{\tilde{z}}{\times} \mathbb{1} \right) \mathbb{1}^J_z$$

$${}_M \mathbb{1} \sum_I \mathbb{1}^I \mathbb{1}^{\tilde{z}^J} = \sum_I {}_M \delta^I \mathbb{1}^{\tilde{z}^J} = {}_M \mathbb{1}^{\tilde{z}^J} = {}_M \mathbb{1} \overset{\tilde{z}}{\times} \mathbb{1}^J$$

$${}_M \mathbb{1} \mathbb{1}^J = {}_M \delta^J = \det_M \left(\mathbb{1}^z \mathbb{1} \right)^J = \sum_{|I|=|J|} {}_M \mathbb{1}^I \mathbb{1}^z \mathbb{1}^J = \sum_I ({}_M \mathbb{1} \times \mathbb{1} \mathbb{1})^z \mathbb{1}^J$$

$$\overset{\tilde{z}}{\times} \mathbb{1}^I = \mathbb{1}^I \eta^I$$

$$\times \mathbb{1}^I = ({}_I \mathbb{1}) \eta^I$$

$$\mathbb{1}^{\mathbb{K}} \ni \begin{cases} \mathbb{1}^J = \mathbb{1}^J \mathbb{1} \\ \mathbb{1}^J = \sum_{j \in J} \mathbb{1}^j = \mathbb{1} \mathbb{1}^J \end{cases} \text{ dual ONBasis}$$

$$\begin{cases} \mathbb{1} = \mathbb{1}^I \mathbb{1} \\ \mathbb{1} = \mathbb{1}^I \mathbb{1} \end{cases}$$

$$\begin{cases} \mathbb{1}^I \times \mathbb{1}^J = \mathbb{1}^I \mathbb{1} \mathbb{1}^J = \mathbb{1}^I \underbrace{\mathbb{1} \mathbb{1}^J}_{\mathbb{1}} = \mathbb{1}^I \mathbb{1} \mathbb{1}^J = \mathbb{1}^I \mathbb{1} \mathbb{1}^J & = \mathbb{1}^I \mathbb{1} \mathbb{1}^J \\ \mathbb{1}^I \times \mathbb{1}^J = \det \mathbb{1}^i \left(\mathbb{1} \eta \mathbb{1} \right) \mathbb{1}^j = \det \left(\mathbb{1} \mathbb{1}^i \right) \eta \left(\mathbb{1} \mathbb{1}^j \right) = \det \mathbb{1}^i \mathbb{1} \eta \mathbb{1}^j = \mathbb{1}^I \mathbb{1} \mathbb{1}^J = \mathbb{1}^I \underbrace{\mathbb{1} \mathbb{1}^J}_{\mathbb{1}} = \mathbb{1}^I \mathbb{1} \mathbb{1}^J & = \mathbb{1}^I \mathbb{1} \mathbb{1}^J \end{cases}$$

$$\overset{*}{z} \mathbb{1}^I = \mathbb{1}^{N-I} \overline{\mathbb{1}^{\mathbb{K}}} \eta^I$$

$$\overset{*}{z} \mathbb{1}^J = \sum_{|I|=|J|} \mathbb{1}^{N-I} \overline{\mathbb{1}^{\mathbb{K}}} \mathbb{1}^J_z \left(\mathbb{1} \eta^N / \mathbb{1} \mathbb{1}^N \right)^{1/2}$$

$$\mathbb{1}^N = c \mathbb{1}^N$$

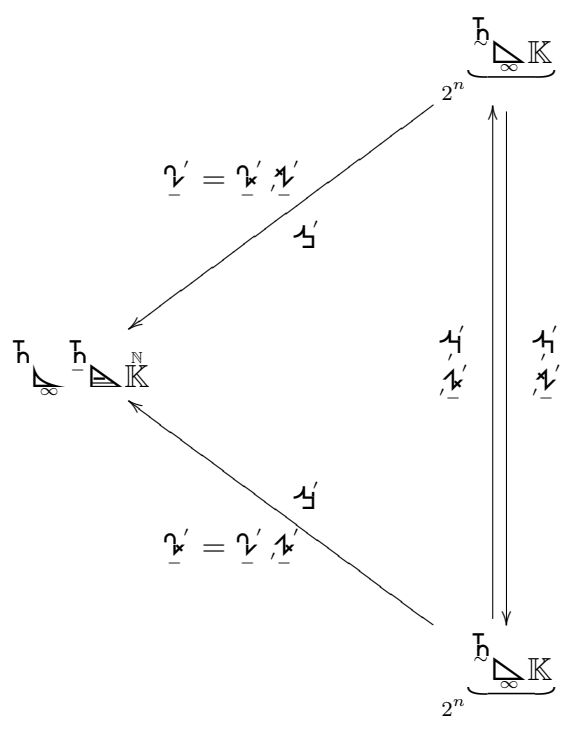
$${}_N \eta^N = \mathbb{1}^N \times \mathbb{1}^N = c^2 \mathbb{1}^N \times \mathbb{1}^N = c^2 \mathbb{1}^N \mathbb{1}^N \Rightarrow \text{LHS} = \sum_I \underbrace{\mathbb{1} \mathbb{1}^N}_{\mathbb{1}} \mathbb{1}^J_z = \sum_I \mathbb{1} \mathbb{1}^N \left(\mathbb{1} \eta^N / \mathbb{1} \mathbb{1}^N \right)^{1/2} \mathbb{1}^J_z = \text{RHS}$$

$$\mathbb{1} = \begin{cases} \mathbb{1} \mathbb{1}^J \\ \mathbb{1} \mathbb{1}^J \end{cases} : \mathbb{1} \delta^J = \begin{cases} \mathbb{1} \mathbb{1}^J \\ \mathbb{1} \mathbb{1}^J \end{cases}$$

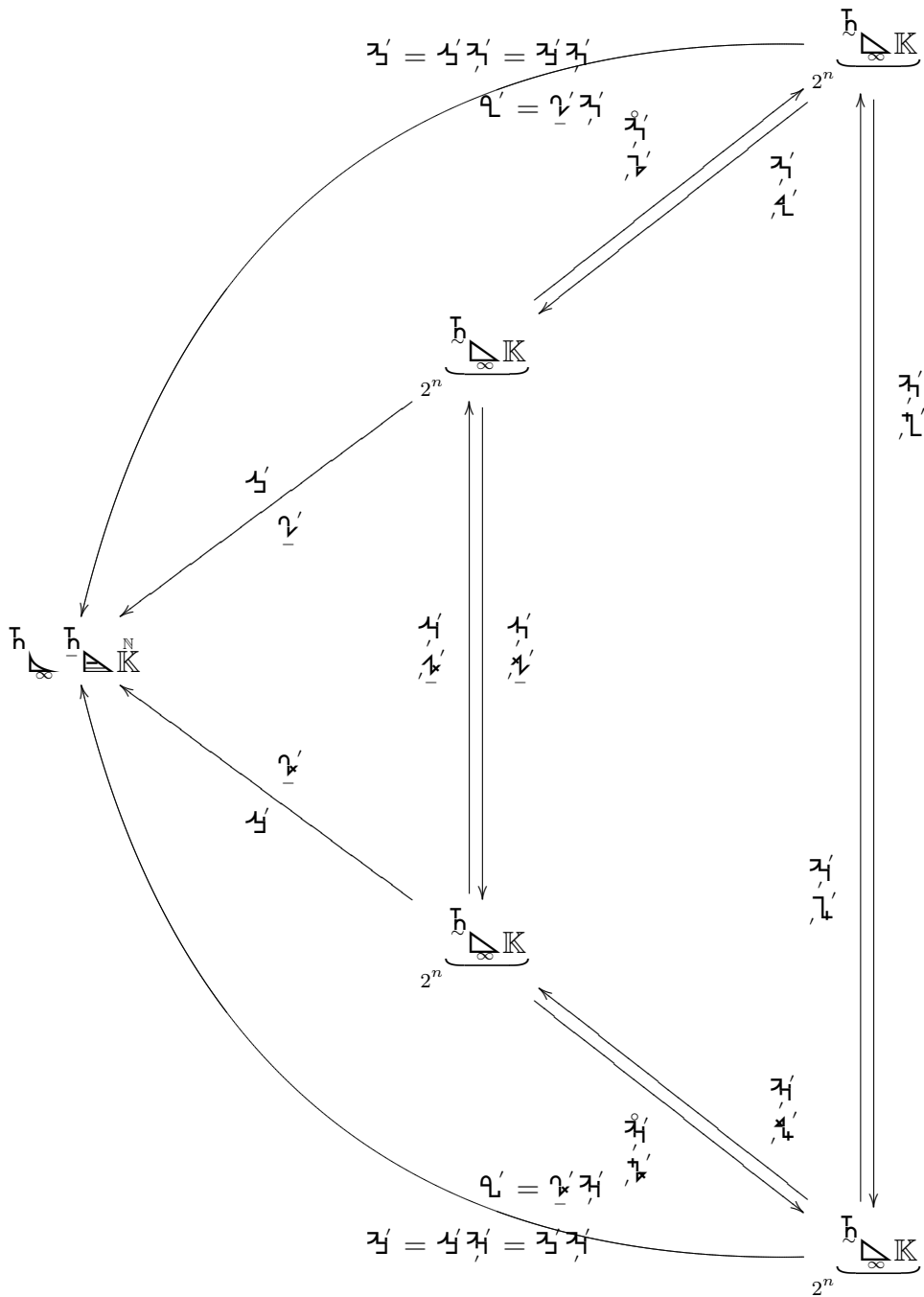
$$\mathbb{1} = \begin{cases} \mathbb{1} \mathbb{1}^N \\ \mathbb{1} \mathbb{1}^N \end{cases} : {}_M \delta^N = \begin{cases} \mathbb{1} \mathbb{1}^N \\ \mathbb{1} \mathbb{1}^N \end{cases}$$

$$\mathbb{1} \mathbb{1} = \begin{cases} \mathbb{1} \mathbb{1}^J = \mathbb{1}^L \mathbb{1}^J \\ \mathbb{1} \mathbb{1}^J = \mathbb{1}^L \mathbb{1}^J \end{cases} \quad \mathbb{1}^J = \begin{cases} \mathbb{1}^L \mathbb{1}^J \\ \mathbb{1}^L \mathbb{1}^J \end{cases}$$

$$\begin{aligned}
\underline{\mathcal{L}}' \cdot \underline{\mathcal{A}} &= \begin{cases} \underline{\mathcal{A}}_1 \underline{\mathcal{A}}'_1 &= \underline{\mathcal{A}}_1 \underline{\mathcal{A}}'_1 \\ \underline{\mathcal{A}}_2 \underline{\mathcal{A}}'_2 &= \underline{\mathcal{A}}_2 \underline{\mathcal{A}}'_2 \end{cases} \quad \underline{\mathcal{L}}^N = \begin{cases} \underline{\mathcal{A}}_1 \underline{\mathcal{A}}^N &= \underline{\mathcal{A}}_1^K \underline{\mathcal{A}}^N \\ \underline{\mathcal{A}}_2 \underline{\mathcal{A}}^N &= \underline{\mathcal{A}}_2^K \underline{\mathcal{A}}^N \end{cases} \\
\begin{cases} \underline{\mathcal{A}}_1 &= \underline{\mathcal{L}}' \underline{\mathcal{A}}'_1 = \underline{\mathcal{A}}_1 \underline{\mathcal{L}}' \\ \underline{\mathcal{A}}_2 &= \underline{\mathcal{L}}' \underline{\mathcal{A}}'_2 = \underline{\mathcal{A}}_2 \underline{\mathcal{L}}' \end{cases} & \begin{cases} \underline{\mathcal{A}}^J &= \underline{\mathcal{L}}^L \underline{\mathcal{A}}^J = \underline{\mathcal{A}} \underline{\mathcal{L}}^J \\ \underline{\mathcal{A}}^J &= \underline{\mathcal{L}}^L \underline{\mathcal{A}}^J = \underline{\mathcal{A}} \underline{\mathcal{L}}^J \end{cases} \\
\begin{cases} \underline{\mathcal{A}}_1 &= \underline{\mathcal{L}}' \underline{\mathcal{A}}'_1 = \underline{\mathcal{A}}_1 \underline{\mathcal{L}}' \\ \underline{\mathcal{A}}_2 &= \underline{\mathcal{L}}' \underline{\mathcal{A}}'_2 = \underline{\mathcal{A}}_2 \underline{\mathcal{L}}' \end{cases} & \begin{cases} \underline{\mathcal{A}}^N &= \underline{\mathcal{L}}^K \underline{\mathcal{A}}^N = \underline{\mathcal{A}} \underline{\mathcal{L}}^N \\ \underline{\mathcal{A}}^N &= \underline{\mathcal{L}}^K \underline{\mathcal{A}}^N = \underline{\mathcal{A}} \underline{\mathcal{L}}^N \end{cases} \\
\begin{cases} \underline{\mathcal{A}}_1 &= \underline{\mathcal{L}} \underline{\mathcal{A}}'_1 = \underline{\mathcal{A}}_1 \underline{\mathcal{L}}' \\ \underline{\mathcal{A}}_2 &= \underline{\mathcal{L}} \underline{\mathcal{A}}'_2 = \underline{\mathcal{A}}_2 \underline{\mathcal{L}}' \end{cases} & \begin{cases} \underline{\mathcal{A}}^J &= \underline{\mathcal{L}}_M \underline{\mathcal{A}}^J = \underline{\mathcal{A}} \underline{\mathcal{L}}^J \\ \underline{\mathcal{A}}^J &= \underline{\mathcal{L}}_M \underline{\mathcal{A}}^J = \underline{\mathcal{A}} \underline{\mathcal{L}}^J \end{cases} \\
\begin{cases} \underline{\mathcal{A}}_1 &= \underline{\mathcal{L}} \underline{\mathcal{A}}'_1 = \underline{\mathcal{A}}_1 \underline{\mathcal{L}}' \\ \underline{\mathcal{A}}_2 &= \underline{\mathcal{L}} \underline{\mathcal{A}}'_2 = \underline{\mathcal{A}}_2 \underline{\mathcal{L}}' \end{cases} & \begin{cases} \underline{\mathcal{A}}^N &= \underline{\mathcal{L}}_I \underline{\mathcal{A}}^N = \underline{\mathcal{A}} \underline{\mathcal{L}}^N \\ \underline{\mathcal{A}}^N &= \underline{\mathcal{L}}_I \underline{\mathcal{A}}^N = \underline{\mathcal{A}} \underline{\mathcal{L}}^N \end{cases}
\end{aligned}$$



$\mathcal{H}_\infty \mathcal{H}_\infty \mathcal{K} \mathcal{L} \mathcal{V}^J$ holonomic basis
 $\underline{\mathcal{A}} = \underline{\mathcal{L}} \underline{\mathcal{V}} \cdot \underline{\mathcal{A}} : \quad M \delta^N = M \underline{\mathcal{L}} \underline{\mathcal{V}}^N$



$$\mathbb{h}_{\infty} \mathbb{h}_{\infty} \mathbb{K} \ni \begin{cases} \mathcal{A}^J \\ \mathcal{A}^J = \sum_{j \in J} \mathcal{A}^j \end{cases} \text{ dual ONbasis}$$

$$\mathcal{A}^I \times_{\mathbb{h}} \mathcal{A}^J = \delta_{I,J}$$

$$\mathbf{x}_I \mathbf{b} = \mathbf{a}_I \eta^I$$

$$\mathbf{a}_I = (\mathbf{x}_I \mathbf{b}) \eta^I$$

$$\ast \mathbf{a}_I = \mathbf{a}^{N-I} \overline{I > \overline{N-I}} \eta^I$$

$$\mathbf{a}_I = \begin{pmatrix} \mathbf{x}_I \mathbf{z}_I \\ \mathbf{b}_I \mathbf{a}_I \end{pmatrix} : \quad \mathbf{a}_I \delta^J = \begin{pmatrix} \mathbf{x}_I \mathbf{z}_I^J \\ \mathbf{b}_I \mathbf{a}_I^J \end{pmatrix}$$

$$\begin{cases} \mathbf{z}_I^J = \mathbf{z}'_I \mathbf{z}'_I^J \\ \mathbf{a}_I^J = \mathbf{a}'_I \mathbf{a}'_I^J \end{cases} \quad \begin{cases} \mathbf{z}_I^J = \mathbf{z}_I^L \mathbf{z}_L^J \\ \mathbf{a}_I^J = \mathbf{a}_I^L \mathbf{a}_L^J \end{cases}$$

$$\mathbf{z}'_I \mathbf{a}'_I = \begin{pmatrix} \mathbf{z}'_I \mathbf{z}'_I^N \\ \mathbf{a}'_I \mathbf{b}'_I^N \end{pmatrix} : \quad \mathbf{z}'_I^N = \begin{pmatrix} \mathbf{z}_I^K \mathbf{z}_K^N \\ \mathbf{a}_I^K \mathbf{b}_K^N \end{pmatrix}$$

$$\begin{cases} \mathbf{z}'_I^J = \mathbf{z}_M^J \mathbf{z}_M^I \\ \mathbf{a}'_I^J = \mathbf{a}_M^J \mathbf{a}_M^I \end{cases} \quad \begin{cases} \mathbf{z}_M^J = \mathbf{z}_M^J \\ \mathbf{a}_M^J = \mathbf{a}_M^J \end{cases}$$

$$\begin{cases} \mathbf{z}'_I^N = \mathbf{z}_I^N \\ \mathbf{b}'_I^N = \mathbf{b}_I^N \end{cases} \quad \begin{cases} \mathbf{z}_I^N = \mathbf{z}_I^N \\ \mathbf{b}_I^N = \mathbf{b}_I^N \end{cases}$$

