

$$\mathbb{K}^m \triangleleft_{2^m} \mathbb{K} \xleftarrow{\mathcal{A}} \mathbb{K}^n \triangleleft_{2^n} \mathbb{K} \Leftarrow \mathbb{K}^m \xrightarrow{\mathcal{A}} \mathbb{K}^n$$

$$\underbrace{\mathcal{A} \cdot \mathbb{A}}_I = \sum_{|J|=|I|} \det \mathcal{A}^J \underbrace{\mathcal{A} \cdot \mathbb{A}}_J$$

$$\underbrace{\mathcal{A} \cdot \mathbb{I}^J}_I = \det \mathcal{A}^J$$

$$\mathcal{A} \cdot \mathbb{I}^J = \sum_{|I|=p} \mathbb{I}^I \det \mathcal{A}^J$$

$$\mathcal{A} \cdot \mathbb{A} = \mathcal{A} \cdot \underbrace{\mathbb{I}^J \mathbb{A}}_J = \underbrace{\mathcal{A} \cdot \mathbb{I}^J}_I \underbrace{\mathcal{A} \cdot \mathbb{A}}_J = \sum_{|I|=p} \mathbb{I}^I \det \mathcal{A}^J \underbrace{\mathcal{A} \cdot \mathbb{A}}_J$$

$$\mathcal{A} \cdot \underbrace{\mathcal{A} \cdot \mathbb{A}}_J = \mathcal{A} \cdot \mathcal{A} \cdot \mathbb{A}$$

$$\underbrace{\mathcal{A} \cdot \mathcal{A} \cdot \mathbb{A}}_I = \det \mathcal{A}^J \mathcal{A} \cdot \underbrace{\mathcal{A} \cdot \mathbb{A}}_J = \det \mathcal{A}^J \mathcal{A} \cdot \underbrace{\det \mathcal{A}^K \mathcal{A} \cdot \mathbb{A}}_K$$

$$= \det \mathcal{A}^J \underbrace{\det \mathcal{A} \cdot \mathcal{A}^K}_J \underbrace{\mathcal{A} \cdot \mathbb{A}}_K = \det \underbrace{\mathcal{A} \cdot \mathcal{A} \cdot \mathcal{A}^K}_I \underbrace{\mathcal{A} \cdot \mathbb{A}}_K$$

$$= \det \underbrace{\mathcal{A} \cdot \mathcal{A} \cdot \mathcal{A}^K}_I \underbrace{\mathcal{A} \cdot \mathbb{A}}_K = \underbrace{\det \mathcal{A} \cdot \mathcal{A} \cdot \mathcal{A}^K}_I \cdot \underbrace{\det \mathcal{A} \cdot \mathbb{A}}_K$$

$$\mathcal{A} \cdot \mathbb{A} \cdot \mathbb{A} = \underbrace{\mathcal{A} \cdot \mathbb{A}}_I \cdot \underbrace{\mathcal{A} \cdot \mathbb{A}}_I$$

$$\mathcal{A} \cdot \mathbb{I}^{j_1} \cdot \mathcal{A} \cdot \mathbb{I}^{j_p} = \sum_{i_1 \dots i_p} \mathbb{I}^{i_1} \mathcal{A}^{j_1}_{i_1} \cdot \mathcal{A} \cdot \mathbb{I}^{i_p} \mathcal{A}^{j_p}_{i_p} = \sum_{i_1 \dots i_p \text{ dist}} \mathbb{I}^{i_1} \mathcal{A}^{j_1}_{i_1} \cdot \mathcal{A} \cdot \mathbb{I}^{i_p} \mathcal{A}^{j_p}_{i_p}$$

$$= \sum_{|I|=p} \mathbb{I}^I \sum_{\pi} (-1)^\pi \mathcal{A}^{j_{\pi 1}}_{i_1} \dots \mathcal{A}^{j_{\pi p}}_{i_p} = \sum_{|I|=p} \mathbb{I}^I \det \mathcal{A}^J = \mathcal{A} \cdot \mathbb{I}^J$$