$$\underset{2^m}{\mathbb{K}^m}\triangle\mathbb{K} \xleftarrow{\lightning} \underset{2^n}{\mathbb{K}^n}\triangle\mathbb{K} \Leftarrow \mathbb{K}^m \xrightarrow{\lightning} \mathbb{K}^n$$

$$\underbrace{\lightning \bullet \P}_{I} = \sum_{|J|=|I|} \det \, {}_I\lightning^J \; \underbrace{\lightning \bowtie {}_J\P'}$$

$$\underbrace{\lightning \bullet \mathbb{L}^J}_{I} = \det \, {}_I\lightning^J$$

$$\lightning \; \bullet\mathbb{L}^J = \sum_{|I|=p} \bullet\mathbb{L}^I \det \, {}_I\lightning^J$$

$$\lightning \; \bullet\P = \lightning \; \underbrace{\bullet\mathbb{L}^J {}_J\P} = \underbrace{\lightning \bullet\mathbb{L}^J}\underbrace{\lightning \bowtie {}_J\P} = \sum_{|I|=p} \bullet\mathbb{L}^I \det \, {}_I\lightning^J \; \underbrace{\lightning \bowtie {}_J\P}$$

$$\lightning \; \underbrace{\lightning' \bullet \P} = \underline{\lightning \, \lightning'} \bullet \P$$

$$\underbrace{\lightning \; \overbrace{\lightning' \bullet \P}}_{I} = \det \, {}_I\lightning^J \; \lightning \bowtie \underbrace{\lightning' \bullet \P}_{J} = \det \, {}_I\lightning^J \; \lightning \bowtie \underbrace{\det \, {}_J\lightning'^K \; \overbrace{\lightning' \bowtie {}_K\P}}$$

$$= \det \, {}_I\lightning^J \; \underbrace{\det \, \lightning \bowtie {}_J\lightning'^K} \; \overbrace{\lightning \bowtie \underbrace{\lightning' \bowtie {}_K\P}} = \det \, \underbrace{\lightning \; \lightning \bowtie \lightning'}_{I}{}^K \; \underbrace{\lightning \, \lightning'} \bowtie {}_K\P$$

$$= \det \, \Big| \lightning \, \lightning' \Big|_I^K \; \underbrace{\lightning \, \lightning'} \bowtie {}_K\P = \overbrace{\Big| \lightning \, \lightning' \Big| \bullet \P}_{K}$$

$$\lightning \; \underbrace{\P \boxtimes \P'} = \underbrace{\lightning \, \P} \boxtimes \underbrace{\lightning \, \P'}$$

$$\lightning \; \mathbb{L}^{j_1} \boxtimes \lightning \; \mathbb{L}^{j_p} = \sum_{i_1 \cdots i_p} \mathbb{L}^{i_1} \; {}_{i_1}\lightning^{j_1} \boxtimes \mathbb{L}^{i_p} \; {}_{i_p}\lightning^{j_p} = \sum_{i_1 \cdots i_p \text{ dist}} \mathbb{L}^{i_1} \; {}_{i_1}\lightning^{j_1} \boxtimes \mathbb{L}^{i_p} \; {}_{i_p}\lightning^{j_p}$$

$$= \sum_{|I|=p} \mathbb{L}^I \sum_{\pi} (-1)^{\pi} \; {}_{i_1}\lightning^{j_{\pi 1}} \cdots {}_{i_p}\lightning^{j_{\pi p}} = \sum_{|I|=p} \mathbb{L}^I \det \, {}_I\lightning^J = \lightning \; \mathbb{L}^J$$