$$\underline{\underline{h}}\mathbb{R}^n \xleftarrow{\quad e^i \quad} \mathbb{R}$$

$$\underline{\underline{h}}\mathbb{R}^n \ni e^j \quad \text{dual standard basis}$$

$$e^i \bullet e^j = \overset{*}{e}{}^i \circ e^j = {}_i e \circ e^j = \eta^{ij}$$

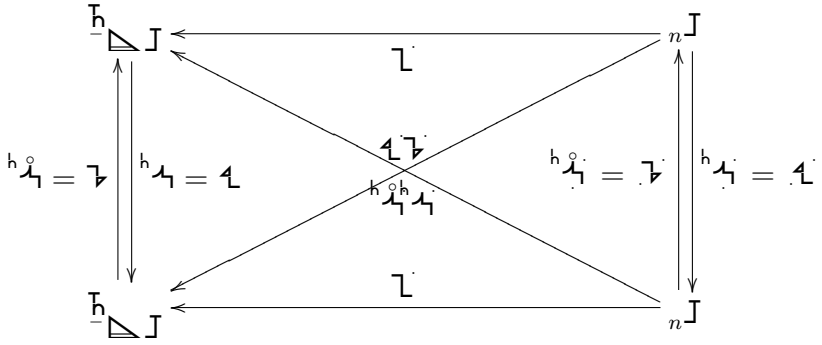
$$v^i = {}_i e \circ \underbrace{e^j \cdot v^i}$$

$${}_i e \circ e^j = {}_i\delta^j$$

$$v^i = {}_i e \circ \underbrace{e^j \cdot v^i}$$

$$\overset{*}{e}{}^i \circ e^j = {}_i\delta^j = {}_i e \circ e^j$$

$$\overset{*}{e}{}^i = {}_i e$$



$$e^i \underset{h}{\bullet} e^j = \begin{cases} \overset{*}{e}{}^i \circ {}^h e_i \circ e^j & = {}^h g^{ij} \\ \overset{*}{e}{}^i \circ {}^h\mathbb{P}_h \circ e^j & = {}_i e \circ {}^h\mathbb{P}_h \circ e^j = {}_i{}^h\mathbb{P}_h^j \end{cases}$$

$$\underline{\underline{h}}\mathbb{R}^n \ni \begin{cases} {}^h e_i{}^j & = {}^h e_i \circ e^j \\ {}^h e^j & = {}^h e \circ e^j \end{cases} \quad \text{dual ONBasis}$$

$$\begin{cases} {}^h e_i & = {}^h e_i{}^j \, {}_j e \\ {}^h e & = {}^h e^j \, {}_j e \end{cases}$$

$$\begin{cases} {}^h e_i \underset{h}{\bullet} {}^h e_j = {}^h \overset{*}{e}{}_i \, {}^h g \, {}^h e_j = {}^h \overset{*}{e}{}_i \, \underbrace{{}^h \bar{e}^* \circ {}^h e} \, {}^h e_j = \overbrace{{}^h e_i \, {}^h \mathring{e}} \circ \underbrace{{}^h e_j \, {}^h \mathring{e}} & = \overbrace{{}^h e \circ e^i} \, \underbrace{{}^h \bar{e}^* \circ {}^h e} \, \underbrace{{}^h e \circ e^j} = {}_i e \underbrace{{}^h \overset{*}{e} \, {}^h \bar{e}^* \circ {}^h e \, {}^h e} \circ e^j \\ e^i \underset{h}{\bullet} e^j = \overset{*}{e}{}^i \circ {}^h\mathbb{P}_h \circ e^j = \overset{*}{e}{}^i \circ \underbrace{\overset{*}{h} \circ h} \circ e^j = \underbrace{\overset{*}{e} \, {}^i \circ h} \circ e^j & = \overbrace{e \circ e^i} \, \underbrace{\overset{*}{h} \circ h} \, \underbrace{e \circ e^j} = {}_i e \underbrace{e \circ \overset{*}{h} \circ h \circ e} \circ e^j \end{cases} = \overset{*}{e}{}_i \circ \eta \circ e^j = \eta$$

$$v \underset{h}{\bullet} v' = \underbrace{e \cdot v} \underset{h}{\bullet} \underbrace{e \cdot v'} = \overbrace{e \cdot v} \circ \overbrace{e \cdot v'} = \overset{*}{e} \cdot v \circ e \cdot v' = \overset{*}{e} \cdot v \circ \mathbb{P}_h \cdot v'$$

$$v_h = \bar{e}^*_h \eta^{\cdot\cdot} e_{\cdot h} = \bar{e}^*_i \eta^{ij} e_{j h}$$

$$\delta^{\cdot}_{\cdot h} = \begin{cases} e_{\cdot h} e^{\cdot}_{\cdot h} \\ v^{\cdot} u_{\cdot h} \end{cases}$$

$${}_i\delta^j = \begin{cases} e_{i h} e^{j h} \\ {}_i v u^j \end{cases}$$

$$\delta^{\cdot}_{\cdot h} = \begin{cases} e^{\cdot}_{\cdot h} e^{\cdot}_{\cdot h} \\ u^{\cdot} v_{\cdot h} \end{cases}$$

$${}_i\delta^j = \begin{cases} e_{i h} e^{j h} \\ {}_i u v^j \end{cases}$$

$$\delta^{\cdot}_{\cdot h} = \begin{cases} e^{\cdot}_{\cdot h} e^{\cdot}_{\cdot h} & = e^{\cdot}_{\cdot h} e^{\cdot}_{\cdot h} \\ v^{\cdot} u_{\cdot h} & = u^{\cdot} v_{\cdot h} \end{cases}$$

$$\delta^j = \begin{cases} e^{\cdot}_{\cdot h} e^{j h} & = e^{\cdot}_{\cdot h} e^{j h}_{k} \\ v u^j & = u^k {}_k v^j \end{cases}$$

$$\delta^{\cdot}_{\cdot h} = \begin{cases} e_{\cdot h} e^{\cdot}_{\cdot h} & = e_{\cdot h} e^{\cdot}_{\cdot h} \\ u^{\cdot} v_{\cdot h} & = u^{\cdot} v_{\cdot h} \end{cases}$$

$$\delta^j = \begin{cases} e_{\cdot h} e^{j h} & = e_{\cdot h} e^{j h}_{k} \\ u v^j & = u^k {}_k u^j \end{cases}$$

$$\begin{cases} e_{\cdot h} & = \delta^{\cdot}_{\cdot} e_{\cdot h} = e_{\cdot h} \delta^{\cdot}_{\cdot} \\ u^{\cdot} & = \delta^{\cdot}_{\cdot} u^{\cdot} = u^{\cdot} \delta^{\cdot}_{\cdot} \end{cases}$$

$$\begin{cases} e_h & = \delta^k e_{k h} = e_h \delta^j \\ u^j & = \delta^k {}_k u^j = u^j \delta^j \end{cases}$$

$$\begin{cases} e^{\cdot}_{\cdot h} & = \delta^{\cdot}_{\cdot} e^{\cdot}_{\cdot h} = e^{\cdot}_{\cdot h} \delta^{\cdot}_{\cdot} \\ v^{\cdot} & = \delta^{\cdot}_{\cdot} v^{\cdot} = v^{\cdot} \delta^{\cdot}_{\cdot} \end{cases}$$

$$\begin{cases} e^{j}_{h} & = \delta^k e^{j}_{k h} = e^{j}_{h} \delta^j \\ v^j & = \delta^k {}_k v^j = v^j \delta^j \end{cases}$$

$$\begin{cases} e_{\cdot h} & = \delta^{\cdot}_{\cdot} e_{\cdot h} = e_{\cdot h} \delta^{\cdot}_{\cdot} \\ u^{\cdot} & = \delta^{\cdot}_{\cdot} u^{\cdot} = u^{\cdot} \delta^{\cdot}_{\cdot} \end{cases}$$

$$\begin{cases} {}^{h}_{i}\text{ग}^{j} & = {}_{i}\text{ग}^{h}\text{ग}^{j} = {}^{h}_{i}\text{गग}^{j} \\ {}_{i}\text{ह}^{j} & = {}_{i}\text{गह}^{j} = {}_{i}\text{हग}^{j} \end{cases}$$

$$\begin{cases} {}^{h}\dot{\text{ग}}.\text{श} & = \dot{\text{ग}}\underbrace{{}^{h}\dot{\text{ग}}.\text{श}} = {}^{h}\dot{\text{ग}}\underbrace{\dot{\text{ग}}.\text{श}} \\ \dot{\text{ह}}.\text{श} & = \dot{\text{ग}}\underbrace{\dot{\text{ह}}.\text{श}} = \dot{\text{ह}}\underbrace{\dot{\text{ग}}.\text{श}} \end{cases}$$

$$\begin{cases} {}^{h}_{i}\text{ङ}^{j} & = {}_{i}\text{ग}^{h}\text{ङ}^{j} = {}^{h}_{i}\text{गग}^{j} \\ {}_{i}\text{ब}^{j} & = {}_{i}\text{गब}^{j} = {}_{i}\text{बग}^{j} \end{cases}$$