$$
{}^{z}\begin{array}{c|c} a & b \\ \hline 0 & d \end{array} \;=\; a^{-1}\underbrace{b + zd}
$$

$$
\frac{{}^{J}b}{{}^{P-J}d} = \begin{array}{c|c|c|c}
{}^{1}b_{1} & {}^{1}b_{j} & {}^{1}b_{j+1} & {}^{1}b_{p} \\ \hline
{}^{j}b_{1} & {}^{j}b_{j} & {}^{j}b_{j+1} & {}^{j}b_{p} \\ \hline
1 & {}^{1}d_{j} & {}^{1}d_{j+1} & {}^{1}d_{p} \\ \hline
0 & 1 & {}^{p-j}d_{j+1} & {}^{p-j}d_{p}
\end{array}
$$

$$
\begin{array}{c|c|c|c}
{}^{1}b_{1} & {}^{1}b_{j} & {}^{1}b_{j+1} & {}^{1}b_{p} \\ \hline
{}^{j}b_{1} & {}^{j}b_{j} & {}^{j}b_{j+1} & {}^{j}b_{p} \\ \hline
{}^{j+1}b_{1} + \lambda_{j+1} & {}^{j+1}b_{j} + \lambda_{j+1}\,{}^{1}d_{j} & {}^{j+1}b_{j+1} + \lambda_{j+1}\,{}^{1}d_{j+1} & {}^{j+1}b_{p} + \lambda_{j+1}\,{}^{1}d_{p} \\ \hline
{}^{p}b_{1} & {}^{p}b_{j} + \lambda_{p}1 & {}^{p}b_{j+1} + \lambda_{p}\,{}^{p-j}d_{j+1} & {}^{p}b_{p} + \lambda_{p}\,{}^{p-j}d_{p}
\end{array}
$$

$$
= \begin{array}{c|c|c|c}
{}^{1}b_{1} & {}^{1}b_{j} & {}^{1}b_{j+1} & {}^{1}b_{p} \\ \hline
{}^{j}b_{1} & {}^{j}b_{j} & {}^{j}b_{j+1} & {}^{j}b_{p} \\ \hline
{}^{j+1}b_{1} & {}^{j+1}b_{j} & {}^{j+1}b_{j+1} & {}^{j+1}b_{p} \\ \hline
{}^{p}b_{1} & {}^{p}b_{j} & {}^{p}b_{j+1} & {}^{p}b_{p}
\end{array}
+
\begin{array}{c|c|c|c}
0 & 0 & 0 & 0 \\ \hline
0 & 0 & 0 & 0 \\ \hline
\lambda_{j+1} & 0 & 0 & 0 \\ \hline
0 & \lambda_{p} & 0 & 0
\end{array}
\begin{array}{c|c|c|c}
1 & {}^{1}d_{j} & {}^{1}d_{j+1} & {}^{1}d_{p} \\ \hline
0 & 1 & {}^{p-j}d_{j+1} & {}^{p-j}d_{p} \\ \hline
0 & 0 & 1 & {}^{p-j+1}d_{p} \\ \hline
0 & 0 & 0 & 1
\end{array}
$$

$$
\begin{array}{c|c} a & b \\ \hline c & d \end{array} =
\begin{array}{ccc|ccc}
{}^{1}a_{1} & {}^{1}a_{j} & {}^{1}a_{q} & {}^{1}b_{p} & {}^{1}b_{k} & {}^{1}b_{1} \\
0 & {}^{i}a_{j} & {}^{i}a_{q} & {}^{i}b_{p} & {}^{i}b_{k} & {}^{i}b_{1} \\
0 & 0 & {}^{q}a_{q} & {}^{q}b_{p} & {}^{q}b_{k} & {}^{q}b_{1} \\ \hline
 & & & {}^{p}d_{p} & {}^{p}d_{k} & {}^{p}d_{1} \\
 & 0 & & 0 & {}^{h}d_{k} & {}^{h}d_{1} \\
 & & & 0 & 0 & {}^{1}d_{1}
\end{array}
$$

$$\underbrace{{}^{1}e\ddot{\times}^{p}e \times {}^{1}u\ddot{\times}^{j}u} \times \overbrace{\underbrace{{}^{1}e\ddot{\times}^{p}e} \vdash \underbrace{{}^{1}u\ddot{\times}^{q}u \times {}^{1}v\ddot{\times}^{p-j}v}} = \det a \frac{{}^{J}b}{{}^{P\llcorner J}d}$$

$$\underbrace{{}^{1}e\ddot{\times}^{p}e} \vdash \underbrace{{}^{1}u\ddot{\times}^{q}u \times {}^{1}v\ddot{\times}^{p-j}v} = \underbrace{{}^{1}e\ddot{\times}^{p}e} \vdash \underbrace{{}^{1}w\ddot{\times}^{p+q-j}w} = \sum_{\nu_1 < \cdot\cdot < \nu_p} (-1)^{\nu} \underbrace{{}^{1}e\ddot{\times}^{p}e|^{\nu_1}w\ddot{\times}^{\nu_p}w}\, {}^{\acute{\nu}_1}w \;\ddot{\times}\; {}^{\acute{\nu}_{q-j}}w$$

$$\underbrace{{}^{1}u\ddot{\times}^{j}u} \times \overbrace{\underbrace{{}^{1}e\ddot{\times}^{p}e} \vdash \underbrace{{}^{1}u\ddot{\times}^{q}u \times {}^{1}v\ddot{\times}^{p-j}v}} = \sum_{\nu_1 < \cdot\cdot < \nu_p} (-1)^{\nu} \underbrace{{}^{1}e\ddot{\times}^{p}e|^{\nu_1}w\ddot{\times}^{\nu_p}w}\, {}^{1}u \;\ddot{\times}\; {}^{j}u \times {}^{\acute{\nu}_1}w \;\ddot{\times}\; {}^{\acute{\nu}_{q-j}}w$$

$$= \sum_{\acute{\nu}_1 > j} (-1)^{\nu} \underbrace{{}^{1}e\ddot{\times}^{p}e|^{\nu_1}w\ddot{\times}^{\nu_p}w}\, {}^{1}u \;\ddot{\times}\; {}^{j}u \times {}^{\acute{\nu}_1}w \;\ddot{\times}\; {}^{\acute{\nu}_{q-j}}w$$

$$\acute{\nu}_1 \cdot\cdot \acute{\nu}_{q-j} \subset \left\{ j+1 \cdot\cdot p : p+1 \cdot\cdot q : \bar{1} \cdot\cdot \overline{p-j} \right\}$$

$$\underbrace{{}^{1}e\ddot{\times}^{p}e \times {}^{1}u\ddot{\times}^{j}u} \times \overbrace{\underbrace{{}^{1}e\ddot{\times}^{p}e} \vdash \underbrace{{}^{1}u\ddot{\times}^{q}u \times {}^{1}v\ddot{\times}^{p-j}v}} = \sum_{\acute{\nu}_1 > j} (-1)^{\nu} \underbrace{{}^{1}e\ddot{\times}^{p}e|^{\nu_1}w\ddot{\times}^{\nu_p}w}\, {}^{1}e \;\ddot{\times}\; {}^{p}e \times {}^{1}u \;\ddot{\times}\; {}^{j}u \times {}^{\acute{\nu}_1}w \;\ddot{\times}\; {}^{\acute{\nu}_{q-j}}w$$

$$\text{If } \acute{\nu}_{q-j} = \bar{\ell}: \quad 1 \leqslant \ell \leqslant p-j \Longrightarrow \begin{array}{c|c} {}^{J}a & {}^{J}b \\ {}^{\acute{\nu}_1}x & {}^{\acute{\nu}_1}y \\ 0 & {}^{\ell}d \\ \hline 0 & {}^{P}e \end{array} = \begin{array}{c} {}^{J}a \\ \hline {}^{\acute{\nu}_1}x \\ \hline 0 \end{array} = 0$$

$$\Longrightarrow \acute{\nu}_1 \cdot\cdot \acute{\nu}_{q-j} \subset \left\{ j+1 \cdot\cdot p : p+1 \cdot\cdot q \right\} \Longrightarrow \acute{\nu}_k = j+k \Longrightarrow \begin{cases} \nu_1 = 1 & \nu_j = j \\ \nu_{j+1} = \bar{1} & \nu_p = \overline{p-j} \end{cases}$$

$$\text{LHS} = \underbrace{{}^{1}e\ddot{\times}^{p}e|^{1}u\ddot{\times}^{j}u \times {}^{1}v\ddot{\times}^{p-j}v}\, {}^{1}e \;\ddot{\times}\; {}^{p}e \times {}^{1}u \;\ddot{\times}\; {}^{j}u \times {}^{j+1}u \;\ddot{\times}\; {}^{q}u$$

$$= \underbrace{{}^{1}e\ddot{\times}^{p}e|^{1}b\ddot{\times}^{j}b \times {}^{1}d\ddot{\times}^{p-j}d}\, {}^{1}e \;\ddot{\times}\; {}^{p}e \times {}^{1}u \;\ddot{\times}\; {}^{j}u \times {}^{j+1}u \;\ddot{\times}\; {}^{q}u = \frac{{}^{J}b}{{}^{P\llcorner J}d} \begin{array}{c|c} {}^{Q}a & {}^{Q}b \\ \hline 0 & {}^{P}e \end{array} = \text{RHS}$$

$$\begin{array}{c} {}^{J}a|^{J}b \\ \hline \begin{array}{c|c} {}^{Q}a & {}^{Q}b \\ \hline {}^{P\llcorner J}0 & {}^{P\llcorner J}d \end{array} \dashv 0|^{P}e \\ \hline 0|^{P}e \end{array} = \sum_{J \subset K \subset Q} \sum_{J \subset H \subset P} \det \begin{array}{c} {}^{K}b_{P} \\ {}^{H\llcorner J}d_{P} \end{array} \begin{array}{c} {}^{J}a \\ \hline {}^{Q\llcorner K}a \\ \hline {}^{P\llcorner H}0 \end{array} = \det \begin{array}{c} {}^{J}b_{P} \\ {}^{P\llcorner J}d_{P} \end{array} \frac{{}^{J}a}{{}^{Q\llcorner J}a} = \frac{{}^{J}b}{{}^{P\llcorner J}d} \det a$$

$$
= \frac{\begin{array}{c|c|c|c} {}^{1}b_{p} & {}^{1}b_{j+1} & {}^{1}b_{j} & {}^{1}b_{1} \\ \hline {}^{j}b_{p} & {}^{j}b_{j+1} & {}^{j}b_{j} & {}^{j}b_{1} \end{array}}{\begin{array}{c|c|c|c} {}^{p}d_{p} & {}^{p}d_{j+1} & {}^{p}d_{j} & {}^{p}d_{1} \\ \hline 0 & {}^{j+1}d_{j+1} & {}^{j+1}d_{j} & {}^{j+1}d_{1} \end{array}} = \begin{array}{c|c} {}^{J}b_{P\llcorner J} & {}^{J}b_{J} \\ \hline {}^{P\llcorner J}d_{P\llcorner J} & {}^{P\llcorner J}d_{J} \end{array}
$$

$$
= \begin{array}{c|c} {}^{P\llcorner J}d_{P\llcorner J} & {}^{P\llcorner J}d_{J} \\ \hline {}^{J}b_{P\llcorner J} & {}^{J}b_{J} \end{array} =
$$

$$
= \det {}^{J}b_{J} - {}^{J}b_{P\llcorner J} \, {}^{P\llcorner J}d^{-1}_{P\llcorner J} \, {}^{P\llcorner J}d_{J}
$$