$$z^{-1} = \frac{\mathrm{grad}_z \Delta}{{}^z\Delta}$$

$$\dot{z} \mathbin{\overline{\wedge}} z^{-1} = \dot{z} \mathbin{\overline{\wedge}} \frac{\mathrm{grad}_z \Delta}{{}^z\Delta} = \frac{\dot{z} \mathbin{\overline{\wedge}} \mathrm{grad}_z \Delta}{{}^z\Delta} = \frac{\dot{z}\, {}^z\underset{-}{\Delta}}{{}^z\Delta}$$

$$\dot{z}\, {}^z\underset{-}{\Delta} = \dot{z} \mathbin{\overline{\wedge}} z^{-1}\, {}^z\Delta$$

$$\underbrace{\begin{array}{c|c} 0 & 0 \\ \hline 1 & 0 \end{array}\; {}^0\underset{-}{p}} \mathbin{\overline{\wedge}} {}^0p^{-1} = \frac{\begin{array}{c|c} 0 & 0 \\ \hline 1 & 0 \end{array}\; {}^0\underset{-}{p}\, {}^{0p}\underset{-}{\Delta}}{{}^{0p}\Delta} = \frac{\begin{array}{c|c} 0 & 0 \\ \hline 1 & 0 \end{array}\; {}^0\underline{p \ltimes \Delta}}{{}^0\overset{\frown}{p \ltimes \Delta}}$$

$$\dot{z}\, {}^0\underline{\overset{\frown}{qg} \ltimes \Delta} = \dot{z}\, {}^0\underline{q \ltimes (g \ltimes \Delta)} = \dot{z}\, {}^0\underset{-}{q}\, {}^{0q}\underline{g \ltimes \Delta} = \dot{z}\, {}^0\underline{g \ltimes \Delta}$$

$$g = \begin{array}{c|c} a & b \\ \hline c & d \end{array} = \begin{array}{cc|cc} {}^1a_1 & {}^1a_2 & {}^1b_1 & {}^1b_2 \\ {}^2a_1 & {}^2a_2 & {}^2b_1 & {}^2b_2 \\ \hline {}^1c_1 & {}^1c_2 & {}^1d_1 & {}^1d_2 \\ {}^2c_1 & {}^2c_2 & {}^2d_1 & {}^2d_2 \end{array}$$

$$q = \begin{array}{c|c} \alpha & 0 \\ \hline \gamma & \delta \end{array} = \begin{array}{cc|cc} {}^1\alpha_1 & 0 & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ {}^2\alpha_1 & {}^2\alpha_2 & & \\ \hline {}^1\gamma_1 & {}^1\gamma_2 & {}^1\delta_1 & 0 \\ {}^2\gamma_1 & {}^2\gamma_2 & {}^2\delta_1 & {}^2\delta_2 \end{array}$$

$$qg = \begin{array}{c|c} \alpha & 0 \\ \hline \gamma & \delta \end{array}\; \begin{array}{c|c} a & b \\ \hline c & d \end{array} = \begin{array}{c|c} \alpha a & \alpha b \\ \hline \gamma a + \delta c & \gamma b + \delta d \end{array}$$

$$\tilde{g} = \underbrace{{}^1d_1\,{}^1b_2 - {}^1b_1\,{}^1d_2}\det a + \underbrace{{}^1a_1\,{}^1c_2 - {}^1c_1\,{}^1a_2}\det b = \overset{2}{\det}\,{}^0\underline{g}\ \ \mathrm{tr}\ \overbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{g}}\ {}^0\overset{-1}{g}$$

$$\widetilde{qg} = {}^1\alpha_1^2\,{}^2\alpha_2\,{}^1\delta_1\,\tilde{g} = \overset{2}{\det}\,\alpha\,\frac{{}^1\delta_1}{{}^2\alpha_2}\,\tilde{g}$$

$$p = \begin{array}{cc|cc} 1 & {}^1a_2 & {}^1b_1 & {}^1b_2 \\ 0 & 1 & {}^2b_1 & {}^2b_2 \\ \hline 0 & 0 & 1 & {}^1d_2 \\ 0 & 0 & 0 & 1 \end{array}$$

$$\tilde{p} = {}^1b_2 - {}^1b_1\,{}^1d_2$$

$$\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{p} = \overset{-1}{a}\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ d = \begin{array}{c|c}1&-{}^1a_2\\\hline 0&1\end{array}\ \begin{array}{c|c}0&0\\\hline 1&0\end{array}\ \begin{array}{c|c}1&{}^1d_2\\\hline 0&1\end{array} = \begin{array}{c|c}-{}^1a_2&-{}^1a_2\,{}^1d_2\\\hline 1&{}^1d_2\end{array}$$

$${}^0p = \overset{-1}{a}\,b = \begin{array}{c|c}1&-{}^1a_2\\\hline 0&1\end{array}\ \begin{array}{c|c}{}^1b_1&{}^1b_2\\\hline {}^2b_1&{}^2b_2\end{array} = \begin{array}{c|c}{}^1b_1 - {}^1a_2\,{}^2b_1&{}^1b_2 - {}^1a_2\,{}^2b_2\\\hline {}^2b_1&{}^2b_2\end{array}$$

$$\underbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{p}}\ {}^0\overset{-1}{p} = \begin{array}{c|c}-{}^1a_2&-{}^1a_2\,{}^1d_2\\\hline 1&{}^1d_2\end{array}\ \begin{array}{c|c}{}^2b_2&{}^1a_2\,{}^2b_2 - {}^1b_2\\\hline -{}^2b_1&{}^1b_1 - {}^1a_2\,{}^2b_1\end{array} = \frac{\det a}{\det b}\underbrace{{}^1b_1\,{}^1d_2 - {}^1b_2}$$

$$\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{q} = \begin{array}{c|c}{}^1\alpha_1&0\\\hline {}^2\alpha_1&{}^2\alpha_2\end{array}^{-1}\ \begin{array}{c|c}0&0\\\hline 1&0\end{array}\ \begin{array}{c|c}{}^1\delta_1&0\\\hline {}^2\delta_1&{}^2\delta_2\end{array} = \frac{{}^1\delta_1}{{}^2\alpha_2}\begin{array}{c|c}0&0\\\hline 1&0\end{array}$$

$$\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{qg} = \begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{q}\ {}^0\underline{g} = \frac{{}^1\delta_1}{{}^2\alpha_2}\underbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{g}}$$

$${}^0\overset{\frown}{qg} = {}^0g$$

$$\underbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{qg}}\ {}^0\overset{-1}{\overset{\frown}{qg}} = \frac{{}^1\delta_1}{{}^2\alpha_2}\underbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{g}}\ {}^0\overset{-1}{g}$$

$$\mathrm{tr}\ \underbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{qg}}\ {}^0\overset{-1}{\overset{\frown}{qg}} = \frac{{}^1\delta_1}{{}^2\alpha_2}\ \mathrm{tr}\ \underbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{g}}\ {}^0\overset{-1}{g}$$

$$\overset{2}{\det}\,{}^0\underline{qg}\ \ \mathrm{tr}\ \overbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{qg}}\ {}^0\overset{-1}{\overset{\frown}{qg}} = {}^1\alpha_1^2\,{}^2\alpha_2^2\,\frac{{}^1\delta_1}{{}^2\alpha_2}\,\overset{2}{\det}\,{}^0\underline{g}\ \ \mathrm{tr}\ \overbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{g}}\ {}^0\overset{-1}{g} = {}^1\alpha_1^2\,{}^2\alpha_2\,{}^1\delta_1\,\overset{2}{\det}\,{}^0\underline{g}\ \ \mathrm{tr}\ \overbrace{\begin{array}{c|c}0&0\\\hline 1&0\end{array}\ {}^0\underline{g}}\ {}^0\overset{-1}{g}$$