$$
\text{(commutative diagram — top-left object)} \xrightarrow{\quad} \ \in \mathbb{R}^n
$$

$$
\Big\downarrow d \qquad\qquad\qquad\qquad\qquad \Big\downarrow d
$$

$$
\text{(bottom-left object)} \xrightarrow{\quad} \ \mathbb{R}^n
$$

$$
{}_i^{\;j} = {}_i^{\,m}\,{}_m^{\,n}\,{}_n^{\,j} + \bar{}_i^{\,m}\,{}_m^{\,k}
$$

$$
= {}^{-1}
$$

$$
\bar{}_i^{\;j} = {}_i^{\,m}\,\bar{}_m^{\,q}\,{}_q^{\,j}
$$

$$
\bar{} = \ \bar{}\
$$

$$
\partial\times\partial\,\bar{}_i^{\,j} = \partial\times\partial\,\bar{}_i^{\,j} - \partial{}_i^{\,k}\,\partial{}_k^{\,j} + \partial{}_i^{\,k}\,\partial{}_k^{\,j}
$$

$$
= \partial\times\partial\,\underbrace{\bar{}_i^{\,m}\,{}_m^{\,n}\,{}_n^{\,j} + {}_i^{\,m}\,\widehat{d{}_m^{\,n}}\,{}_n^{\,j} - {}_i^{\,m}\,{}_m^{\,n}\,\bar{}_n^{\,j} - \bar{}_i^{\,m}\times\bar{}_m^{\,j}}
$$

$$
-\partial\,\underbrace{{}_i^{\,m}\,{}_m^{\,n}\,{}_n^{\,k} + \bar{}_i^{\,m}\,{}_m^{\,k}}\,\partial\,\underbrace{{}_k^{\,p}\,{}_p^{\,q}\,{}_q^{\,j} - {}_k^{\,p}\,\bar{}_p^{\,j}} + \partial\,\underbrace{{}_i^{\,m}\,{}_m^{\,n}\,{}_n^{\,k} + \bar{}_i^{\,m}\,{}_m^{\,k}}\,\partial\,\underbrace{{}_k^{\,p}\,{}_p^{\,q}\,{}_q^{\,j} - {}_k^{\,p}\,\bar{}_p^{\,j}}
$$

$$
= \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{{}_m^{\,n}}\,{}_n^{\,j} - \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{{}_m^{\,n}}\,{}_n^{\,j} + {}_i^{\,m}\,\underbrace{\partial\times\partial\,\widehat{d{}_m^{\,n}}}\,{}_n^{\,j} - {}_i^{\,m}\,\underbrace{\partial{}_m^{\,n}}\,\underbrace{\partial\,\bar{}_n^{\,j}} + {}_i^{\,m}\,\underbrace{\partial{}_m^{\,n}}\,\underbrace{\partial\,\bar{}_n^{\,j}} - \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{\bar{}_m^{\,j}}
$$

$$
+ \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{\bar{}_n^{\,j}} - {}_i^{\,m}\,\underbrace{\partial{}_m^{\,n}}\,\underbrace{\partial{}_n^{\,q}}\,{}_q^{\,j} - \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{\bar{}_m^{\,j}} + {}_i^{\,m}\,\underbrace{\partial{}_m^{\,n}}\,\underbrace{\partial\,\bar{}_n^{\,j}} + \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{\bar{}_m^{\,j}} + {}_i^{\,m}\,\underbrace{\partial{}_m^{\,n}}\,\underbrace{\partial{}_p^{\,q}}\,{}_q^{\,j}
$$

$$
+ \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{{}_m^{\,q}}\,{}_q^{\,j} - {}_i^{\,m}\,\underbrace{\partial{}_m^{\,n}}\,\underbrace{\partial\,\bar{}_n^{\,j}} - \partial\,\underbrace{\bar{}_i^{\,m}}\,\partial\,\underbrace{\bar{}_m^{\,j}}
$$

$$
= {}_i^{\,m}\,\underbrace{\partial\times\partial\,\widehat{d{}_m^{\,q}} - \partial{}_m^{\,n}\,\partial{}_n^{\,q} + \partial{}_m^{\,n}\,\partial{}_p^{\,q}}\,{}_q^{\,j} = {}_i^{\,m}\,\underbrace{\partial\times\partial\,{}_m\bar{}^{\,q}}\,{}_q^{\,j}
$$