

$$\begin{array}{ccc}
\mathcal{A} \in + \mathbb{H}_{\infty} \mathbb{H} \triangleleft \mathbb{G} | \mathbb{H} & \xrightarrow{\mathcal{A}\mathcal{A}} & + \mathbb{H}_{\infty} \mathbb{H} \triangleleft \mathbb{G}_n \mathbb{R}^n \ni \mathcal{A}\mathcal{A}\mathcal{A} = \mathcal{A} \\
\downarrow d & & \downarrow d \\
\bar{\mathcal{A}} \in \mathbb{H}_{\infty} \mathbb{H} \triangleleft \mathbb{G}^2 | \mathbb{H} & \xrightarrow{\mathcal{A}\mathcal{A}} & \mathbb{H}_{\infty} \mathbb{H} \triangleleft \mathbb{G}_n^2 \mathbb{R}^n \ni \bar{\mathcal{A}}
\end{array}$$

$$\mathcal{A}_i^j = \mathcal{A}_i^m \mathcal{A}_m^n \mathcal{A}_n^j + \bar{\mathcal{A}}_i^m \mathcal{A}_m^k$$

$$\mathcal{A} = \bar{\mathcal{A}}^{-1}$$

$$\bar{\mathcal{A}}_i^j = \mathcal{A}_i^m \bar{\mathcal{A}}_m^q \mathcal{A}_q^j$$

$$\bar{\bar{\mathcal{A}}} = \mathcal{A} \bar{\mathcal{A}} \mathcal{A}$$

$$\begin{aligned}
\mathcal{B} \times \mathcal{C} \bar{\mathcal{A}}_i^j &= \mathcal{B} \times \mathcal{C} \bar{\mathcal{A}}_i^j - \underbrace{\mathcal{B} \mathcal{A}_i^k}_{\mathcal{B} \mathcal{A}_i^k} \underbrace{\mathcal{C} \mathcal{A}_k^j}_{\mathcal{C} \mathcal{A}_k^j} + \underbrace{\mathcal{B} \mathcal{A}_i^k}_{\mathcal{B} \mathcal{A}_i^k} \underbrace{\mathcal{C} \mathcal{A}_k^j}_{\mathcal{C} \mathcal{A}_k^j} \\
&= \mathcal{B} \times \mathcal{C} \left(\overline{\mathcal{A}_i^m \mathcal{A}_m^n \mathcal{A}_n^j} + \mathcal{A}_i^m \overline{d \mathcal{A}_m^n \mathcal{A}_n^j} - \mathcal{A}_i^m \mathcal{A}_m^n \overline{\mathcal{A}_n^j} - \bar{\mathcal{A}}_i^m \times \overline{\mathcal{A}_m^j} \right) \\
&\quad - \underbrace{\mathcal{B} \mathcal{A}_i^m \mathcal{A}_m^n \mathcal{A}_n^k}_{\mathcal{B} \mathcal{A}_i^m \mathcal{A}_m^n \mathcal{A}_n^k} + \underbrace{\bar{\mathcal{A}}_i^m \mathcal{A}_m^k}_{\bar{\mathcal{A}}_i^m \mathcal{A}_m^k} \underbrace{\mathcal{C} \mathcal{A}_k^p \mathcal{A}_p^q \mathcal{A}_q^j}_{\mathcal{C} \mathcal{A}_k^p \mathcal{A}_p^q \mathcal{A}_q^j} - \underbrace{\mathcal{A}_i^p \mathcal{A}_p^q}_{\mathcal{A}_i^p \mathcal{A}_p^q} \underbrace{\bar{\mathcal{A}}_q^j}_{\bar{\mathcal{A}}_q^j} + \underbrace{\mathcal{B} \mathcal{A}_i^m \mathcal{A}_m^n \mathcal{A}_n^k}_{\mathcal{B} \mathcal{A}_i^m \mathcal{A}_m^n \mathcal{A}_n^k} + \underbrace{\bar{\mathcal{A}}_i^m \mathcal{A}_m^k}_{\bar{\mathcal{A}}_i^m \mathcal{A}_m^k} \underbrace{\mathcal{C} \mathcal{A}_k^p \mathcal{A}_p^q \mathcal{A}_q^j}_{\mathcal{C} \mathcal{A}_k^p \mathcal{A}_p^q \mathcal{A}_q^j} - \underbrace{\mathcal{A}_i^p \mathcal{A}_p^q}_{\mathcal{A}_i^p \mathcal{A}_p^q} \underbrace{\bar{\mathcal{A}}_q^j}_{\bar{\mathcal{A}}_q^j} \\
&= \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \mathcal{A}_m^n}_{\mathcal{C} \mathcal{A}_m^n} \mathcal{A}_n^j - \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \mathcal{A}_m^n}_{\mathcal{C} \mathcal{A}_m^n} \mathcal{A}_n^j + \mathcal{A}_i^m \underbrace{\mathcal{B} \times \mathcal{C} \overline{d \mathcal{A}_m^n}}_{\mathcal{B} \times \mathcal{C} \overline{d \mathcal{A}_m^n}} \mathcal{A}_n^j - \mathcal{A}_i^m \underbrace{\mathcal{B} \mathcal{A}_m^n}_{\mathcal{B} \mathcal{A}_m^n} \underbrace{\mathcal{C} \bar{\mathcal{A}}_n^j}_{\mathcal{C} \bar{\mathcal{A}}_n^j} + \mathcal{A}_i^m \underbrace{\mathcal{C} \mathcal{A}_m^n}_{\mathcal{C} \mathcal{A}_m^n} \underbrace{\mathcal{B} \bar{\mathcal{A}}_n^j}_{\mathcal{B} \bar{\mathcal{A}}_n^j} - \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \bar{\mathcal{A}}_m^j}_{\mathcal{C} \bar{\mathcal{A}}_m^j} \\
&\quad + \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \bar{\mathcal{A}}_n^j}_{\mathcal{C} \bar{\mathcal{A}}_n^j} - \mathcal{A}_i^m \underbrace{\mathcal{B} \mathcal{A}_m^n}_{\mathcal{B} \mathcal{A}_m^n} \underbrace{\mathcal{C} \mathcal{A}_n^q}_{\mathcal{C} \mathcal{A}_n^q} \mathcal{A}_q^j - \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \bar{\mathcal{A}}_m^j}_{\mathcal{C} \bar{\mathcal{A}}_m^j} + \mathcal{A}_i^m \underbrace{\mathcal{B} \mathcal{A}_m^n}_{\mathcal{B} \mathcal{A}_m^n} \underbrace{\mathcal{C} \bar{\mathcal{A}}_n^j}_{\mathcal{C} \bar{\mathcal{A}}_n^j} + \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \bar{\mathcal{A}}_m^j}_{\mathcal{C} \bar{\mathcal{A}}_m^j} + \mathcal{A}_i^m \underbrace{\mathcal{B} \mathcal{A}_m^n}_{\mathcal{B} \mathcal{A}_m^n} \underbrace{\mathcal{C} \mathcal{A}_p^q}_{\mathcal{C} \mathcal{A}_p^q} \mathcal{A}_q^j \\
&\quad + \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \mathcal{A}_m^q}_{\mathcal{C} \mathcal{A}_m^q} \mathcal{A}_q^j - \mathcal{A}_i^m \underbrace{\mathcal{B} \mathcal{A}_m^n}_{\mathcal{B} \mathcal{A}_m^n} \underbrace{\mathcal{C} \bar{\mathcal{A}}_n^j}_{\mathcal{C} \bar{\mathcal{A}}_n^j} - \underbrace{\mathcal{B} \bar{\mathcal{A}}_i^m}_{\mathcal{B} \bar{\mathcal{A}}_i^m} \underbrace{\mathcal{C} \bar{\mathcal{A}}_m^j}_{\mathcal{C} \bar{\mathcal{A}}_m^j} \\
&= \mathcal{A}_i^m \underbrace{\mathcal{B} \times \mathcal{C} \overline{d \mathcal{A}_m^q}}_{\mathcal{B} \times \mathcal{C} \overline{d \mathcal{A}_m^q}} - \underbrace{\mathcal{B} \mathcal{A}_m^n}_{\mathcal{B} \mathcal{A}_m^n} \underbrace{\mathcal{C} \mathcal{A}_n^q}_{\mathcal{C} \mathcal{A}_n^q} + \underbrace{\mathcal{B} \mathcal{A}_m^n}_{\mathcal{B} \mathcal{A}_m^n} \underbrace{\mathcal{C} \mathcal{A}_p^q}_{\mathcal{C} \mathcal{A}_p^q} \mathcal{A}_q^j = \mathcal{A}_i^m \underbrace{\mathcal{B} \times \mathcal{C} \bar{\mathcal{A}}_m^q}_{\mathcal{B} \times \mathcal{C} \bar{\mathcal{A}}_m^q} \mathcal{A}_q^j
\end{aligned}$$