$$\frac{a \mid b}{c \mid d} = \begin{array}{ccc|ccc} {}^{1}a_{1} & {}^{1}a_{j} & {}^{1}a_{q} & {}^{1}b_{p} & {}^{1}b_{k} & {}^{1}b_{1} \\ {}^{i}a_{1} & {}^{i}a_{j} & {}^{i}a_{q} & {}^{i}b_{p} & {}^{i}b_{k} & {}^{i}b_{1} \\ {}^{q}a_{1} & {}^{q}a_{j} & {}^{q}a_{q} & {}^{q}b_{p} & {}^{q}b_{k} & {}^{q}b_{1} \\ \hline {}^{p}c_{1} & {}^{p}c_{j} & {}^{p}c_{q} & {}^{p}d_{p} & {}^{p}d_{k} & {}^{p}d_{1} \\ {}^{h}c_{1} & {}^{h}c_{j} & {}^{h}c_{q} & {}^{h}d_{p} & {}^{h}d_{k} & {}^{h}d_{1} \\ {}^{1}c_{1} & {}^{1}c_{j} & {}^{1}c_{q} & {}^{1}d_{p} & {}^{1}d_{k} & {}^{1}d_{1} \end{array}$$

$$\mathbb{C}_{j} \mathbin{⤫} 0_{p+q-j} \frac{a \mid b}{c \mid d} = \begin{array}{c} {}^{1}a \mid {}^{1}b \\ \hline {}^{j}a \mid {}^{j}b \end{array} = {}^{J}a \mid {}^{J}b$$

$$\mathbb{C}_{p+q-j} \mathbin{⤫} 0_{j} \frac{a \mid b}{c \mid d} = \begin{array}{c} {}^{Q}a \mid {}^{Q}b \\ \hline {}^{P \llcorner J}c \mid {}^{P \llcorner J}d \end{array}$$

$$ {}^{R}v \dashv {}^{P}u = \sum \det {}^{L}v \mid {}^{P}u \; {}^{R \llcorner L}v$$

$$ {}^{R}v \mid {}^{R}w \dashv 0 \mid {}^{P}e = \sum \det {}^{L}w_{P} \; {}^{R \llcorner L}v \mid {}^{R \llcorner L}w$$

$$\frac{\begin{array}{c}{}^{J}a\,|\,{}^{J}b\end{array}}{\begin{array}{c|c}{}^{Q}a & {}^{Q}b\\ \hline {}^{P\llcorner J}c & {}^{P\llcorner J}d\end{array}\ \dashv 0|^{P}e} = \sum_{J\subset K\subset Q}\sum_{J\subset H\subset P}\frac{\dfrac{{}^{J}a}{{}^{Q\llcorner K}a}\ {}^{K}b_{P}}{{}^{P\llcorner H}c\qquad {}^{H\llcorner J}d_{P}}$$

$$\frac{\begin{array}{c|c}{}^{Q}a & {}^{Q}b\\ \hline {}^{P\llcorner J}c & {}^{P\llcorner J}d\end{array}\ \dashv 0|^{P}e = \sum_{K\subset Q}\sum_{J\subset H\subset P}\frac{\begin{array}{c|c}{}^{Q\llcorner K}a & {}^{Q\llcorner K}b\\ \hline {}^{P\llcorner H}c & {}^{P\llcorner H}d\end{array}\ {}^{K}b_{P}}{{}^{H\llcorner J}d_{P}}$$

$$\text{LHS} = \sum_{K\subset Q}\sum_{J\subset H\subset P}\frac{\dfrac{\dfrac{{}^{J}a\,|\,{}^{J}b}{{}^{Q\llcorner K}a\,|\,{}^{Q\llcorner K}b}\ {}^{K}b_{P}}{{}^{P\llcorner H}c\,|\,{}^{P\llcorner H}d}}{{}^{H\llcorner J}d_{P}}{0|^{P}e} = \sum_{J\subset K\subset Q}\sum_{J\subset H\subset P}\frac{\dfrac{\dfrac{{}^{J}a\,|\,{}^{J}b}{{}^{Q\llcorner K}a\,|\,{}^{Q\llcorner K}b}\ {}^{K}b_{P}}{{}^{P\llcorner H}c\,|\,{}^{P\llcorner H}d}}{{}^{H\llcorner J}d_{P}}{0|^{P}e} = \text{RHS}$$

$$\frac{\alpha\ \big|\ 0}{\gamma\ \big|\ \delta} = \begin{array}{ccc|c}{}^{1}\alpha_{1} & 0 & 0 & \\ {}^{i}\alpha_{1} & {}^{i}\alpha_{i} & 0 & 0\\ {}^{q}\alpha_{1} & {}^{q}\alpha_{j} & {}^{q}\alpha_{q} & \\ \hline {}^{p}\gamma_{1} & {}^{p}\gamma_{j} & {}^{p}\gamma_{q} & {}^{p}\delta_{p}\ 0\ 0\\ {}^{h}\gamma_{1} & {}^{h}\gamma_{j} & {}^{h}\gamma_{q} & {}^{h}\delta_{p}\ {}^{h}\delta_{h}\ 0\\ {}^{1}\gamma_{1} & {}^{1}\gamma_{j} & {}^{1}\gamma_{q} & {}^{1}\delta_{p}\ {}^{1}\delta_{k}\ {}^{1}\delta_{1}\end{array}$$

$$\mathbb{C}_{j}\times 0_{p+q-j}\frac{\alpha\ \big|\ 0}{\gamma\ \big|\ \delta} = \mathbb{C}_{j}\times 0_{p+q-j}$$

$${}^{J}(\alpha a) = \frac{{}^{1}\alpha_{1}\,{}^{1}a}{{}^{2}\alpha_{1}\,{}^{1}a + {}^{2}\alpha_{2}\,{}^{2}a}{{}^{j}\alpha_{1}\,{}^{1}a \cdot + \cdot + {}^{j}\alpha_{j}\,{}^{j}a}$$

$$n \in Q\llcorner K \Longrightarrow {}^{n}(\alpha a) = {}^{n}\alpha_{1}\,{}^{1}a \cdot + \cdot + {}^{n}\alpha_{n}\,{}^{n}a$$

$$m \in P\llcorner H \Longrightarrow {}^{m}(\gamma a + \delta c) = {}^{m}\gamma_{1}\,{}^{1}a \cdot + \cdot + {}^{m}\gamma_{q}\,{}^{q}a + {}^{m}\delta_{p}\,{}^{p}c \cdot + \cdot + {}^{m}\delta_{m}\,{}^{m}c$$

$$\frac{\dfrac{\dfrac{{}^{1}\alpha_{1}\,{}^{1}a}{{}^{2}\alpha_{1}\,{}^{1}a + {}^{2}\alpha_{2}\,{}^{2}a}}{{}^{j}\alpha_{1}\,{}^{1}a \cdot + \cdot + {}^{j}\alpha_{j}\,{}^{j}a}}{\dfrac{{}^{n}\alpha_{1}\,{}^{1}a \cdot + \cdot + {}^{n}\alpha_{n}\,{}^{n}a}{{}^{m}\gamma_{1}\,{}^{1}a \cdot + \cdot + {}^{m}\gamma_{q}\,{}^{q}a + {}^{m}\delta_{p}\,{}^{p}c \cdot + \cdot + {}^{m}\delta_{m}\,{}^{m}c}}$$

$$= {}^{1}\alpha_{1} .. {}^{j}\alpha_{j}\frac{{}^{J}a}{{}^{n}\alpha_{j+1}\,{}^{j+1}a \cdot + \cdot + {}^{n}\alpha_{n}\,{}^{n}a}{{}^{m}\gamma_{j+1}\,{}^{j+1}a \cdot + \cdot + {}^{m}\gamma_{q}\,{}^{q}a + {}^{m}\delta_{p}\,{}^{p}c \cdot + \cdot + {}^{m}\delta_{m}\,{}^{m}c}$$

$$^{J}(\alpha a)\,|\,^{J}(\alpha b) = {}^{1}\alpha_{1}..{}^{j}\alpha_{j}\;{}^{J}a\,|\,^{J}b$$

$$\frac{{}^{Q}(\alpha a) \mid {}^{Q}(\alpha b)}{{}^{P\llcorner J}(\gamma a+\delta c) \mid {}^{P\llcorner J}(\gamma b+\delta d)} = {}^{1}\alpha_{1}\,{}^{2}\alpha_{2}..{}^{q}\alpha_{q}\,{}^{p}\delta_{p}..{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{j+1}\delta_{j+1}\frac{{}^{Q}a \mid {}^{Q}b}{{}^{P\llcorner J}c \mid {}^{P\llcorner J}d}$$

LHS =

$$\frac{\begin{array}{c}{}^{1}\alpha_{1}\,{}^{1}a\\ {}^{2}\alpha_{1}\,{}^{1}a+{}^{2}\alpha_{2}\,{}^{2}a\\ {}^{q}\alpha_{1}\,{}^{1}a\cdot+\cdot\,{}^{q}\alpha_{q}\,{}^{q}a\end{array} \mid \begin{array}{c}{}^{1}\alpha_{1}\,{}^{1}b\\ {}^{2}\alpha_{1}\,{}^{1}b+{}^{2}\alpha_{2}\,{}^{2}b\\ {}^{q}\alpha_{1}\,{}^{1}b\cdot+\cdot\,{}^{q}\alpha_{q}\,{}^{q}b\end{array}}{\begin{array}{c}{}^{p}\gamma_{1}\,{}^{1}a\cdot+\cdot\,{}^{p}\gamma_{q}\,{}^{q}a+{}^{p}\delta_{p}\,{}^{p}c\\ {}^{p\text{-}1}\gamma_{1}\,{}^{1}a\cdot+\cdot\,{}^{p\text{-}1}\gamma_{q}\,{}^{q}a+{}^{p\text{-}1}\delta_{p}\,{}^{p}c+{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{p\text{-}1}c\\ {}^{j+1}\gamma_{1}\,{}^{1}a\cdot+\cdot\,{}^{j+1}\gamma_{q}\,{}^{q}a+{}^{j+1}\delta_{p}\,{}^{p}c+{}^{j+1}\delta_{p\text{-}1}\,{}^{p\text{-}1}c\cdot+\cdot\,{}^{j+1}\delta_{j+1}\,{}^{j+1}c\end{array} \mid \begin{array}{c}{}^{p}\gamma_{1}\,{}^{1}b\cdot+\cdot\,{}^{p}\gamma_{q}\,{}^{q}b+{}^{p}\delta_{p}\,{}^{p}d\\ {}^{p\text{-}1}\gamma_{1}\,{}^{1}b\cdot+\cdot\,{}^{p\text{-}1}\gamma_{q}\,{}^{q}b+{}^{p\text{-}1}\delta_{p}\,{}^{p}d+{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{p\text{-}1}d\\ {}^{j+1}\gamma_{1}\,{}^{1}b\cdot+\cdot\,{}^{j+1}\gamma_{q}\,{}^{q}b+{}^{j+1}\delta_{p}\,{}^{p}d+{}^{j+1}\delta_{p\text{-}1}\,{}^{p\text{-}1}d\cdot+\cdot\,{}^{j+1}\delta_{j+1}\,{}^{j+1}d\end{array}}$$

$$= {}^{1}\alpha_{1}\,{}^{2}\alpha_{2}..{}^{q}\alpha_{q}$$

$$\frac{\begin{array}{c}{}^{1}a\\ {}^{2}a\\ {}^{q}a\end{array} \mid \begin{array}{c}{}^{1}b\\ {}^{2}b\\ {}^{q}b\end{array}}{\begin{array}{c}{}^{p}\gamma_{1}\,{}^{1}a\cdot+\cdot\,{}^{p}\gamma_{q}\,{}^{q}a+{}^{p}\delta_{p}\,{}^{p}c\\ {}^{p\text{-}1}\gamma_{1}\,{}^{1}a\cdot+\cdot\,{}^{p\text{-}1}\gamma_{q}\,{}^{q}a+{}^{p\text{-}1}\delta_{p}\,{}^{p}c+{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{p\text{-}1}c\\ {}^{j+1}\gamma_{1}\,{}^{1}a\cdot+\cdot\,{}^{j+1}\gamma_{q}\,{}^{q}a+{}^{j+1}\delta_{p}\,{}^{p}c+{}^{j+1}\delta_{p\text{-}1}\,{}^{p\text{-}1}c\cdot+\cdot\,{}^{j+1}\delta_{j+1}\,{}^{j+1}c\end{array} \mid \begin{array}{c}{}^{p}\gamma_{1}\,{}^{1}b\cdot+\cdot\,{}^{p}\gamma_{q}\,{}^{q}b+{}^{p}\delta_{p}\,{}^{p}d\\ {}^{p\text{-}1}\gamma_{1}\,{}^{1}b\cdot+\cdot\,{}^{p\text{-}1}\gamma_{q}\,{}^{q}b+{}^{p\text{-}1}\delta_{p}\,{}^{p}d+{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{p\text{-}1}d\\ {}^{j+1}\gamma_{1}\,{}^{1}b\cdot+\cdot\,{}^{j+1}\gamma_{q}\,{}^{q}b+{}^{j+1}\delta_{p}\,{}^{p}d+{}^{j+1}\delta_{p\text{-}1}\,{}^{p\text{-}1}d\cdot+\cdot\,{}^{j+1}\delta_{j+1}\,{}^{j+1}d\end{array}}$$

$$= {}^{1}\alpha_{1}\,{}^{2}\alpha_{2}..{}^{q}\alpha_{q}$$

$$\frac{\begin{array}{c}{}^{1}a\\ {}^{2}a\\ {}^{q}a\end{array} \mid \begin{array}{c}{}^{1}b\\ {}^{2}b\\ {}^{q}b\end{array}}{\begin{array}{c}{}^{p}\delta_{p}\,{}^{p}c\\ {}^{p\text{-}1}\delta_{p}\,{}^{p}c+{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{p\text{-}1}c\\ {}^{j+1}\delta_{p}\,{}^{p}c+{}^{j+1}\delta_{p\text{-}1}\,{}^{p\text{-}1}c\cdot+\cdot\,{}^{j+1}\delta_{j+1}\,{}^{j+1}c\end{array} \mid \begin{array}{c}{}^{p}\delta_{p}\,{}^{p}d\\ {}^{p\text{-}1}\delta_{p}\,{}^{p}d+{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{p\text{-}1}d\\ {}^{j+1}\delta_{p}\,{}^{p}d+{}^{j+1}\delta_{p\text{-}1}\,{}^{p\text{-}1}d\cdot+\cdot\,{}^{j+1}\delta_{j+1}\,{}^{j+1}d\end{array}}$$

$$= {}^{1}\alpha_{1}\,{}^{2}\alpha_{2}..{}^{q}\alpha_{q}\,{}^{p}\delta_{p}..{}^{p\text{-}1}\delta_{p\text{-}1}\,{}^{j+1}\delta_{j+1}\frac{\begin{array}{c}{}^{1}a\\ {}^{2}a\\ {}^{q}a\end{array} \mid \begin{array}{c}{}^{1}b\\ {}^{2}b\\ {}^{q}b\end{array}}{\begin{array}{c}{}^{p}c\\ {}^{p\text{-}1}c\\ {}^{j+1}c\end{array} \mid \begin{array}{c}{}^{p}d\\ {}^{p\text{-}1}d\\ {}^{j+1}d\end{array}} = \text{RHS}$$

$$\frac{{}^{J}(\alpha a) \mid {}^{J}(\alpha b)}{\dfrac{{}^{Q}(\alpha a) \mid {}^{Q}(\alpha b)}{{}^{P\llcorner J}(\gamma a+\delta c) \mid {}^{P\llcorner J}(\gamma b+\delta d)} \dashv 0 \mid {}^{P}e}$$

$$0 \mid {}^{P}e$$

$$= \underbrace{{}^{1}\alpha_{1}..{}^{j}\alpha_{j}}\,\underbrace{{}^{1}\alpha_{1}..{}^{q}\alpha_{q}}\,\underbrace{{}^{p}\delta_{p}..{}^{j+1}\delta_{j+1}}\,\frac{\begin{array}{c|c} {}^{Q}a & {}^{Q}b \\ \hline {}^{P\llcorner J}c & {}^{P\llcorner J}d \end{array}\ \dashv 0|^{P}e}{0|^{P}e} = \det\alpha \det\delta \frac{{}^{1}\alpha_{1}..{}^{j}\alpha_{j}}{{}^{1}\delta_{1}..{}^{j}\delta_{j}} \frac{\begin{array}{c|c} {}^{Q}a & {}^{Q}b \\ \hline {}^{P\llcorner J}c & {}^{P\llcorner J}d \end{array}\ \dashv 0|^{P}e}{0|^{P}e}$$