$$\psi_3^{\mathbb{R}} = \begin{bmatrix} \psi_3 \\ \tilde{\psi}_3 \end{bmatrix}$$

$$\psi_3 = \begin{bmatrix} \psi_{d_R} & \psi_{e_R} & \psi_{u_R} & \psi_{d_L} & \psi_{e_L} & \psi_{u_L} & \psi_{\nu_L} \end{bmatrix} = \begin{bmatrix} \psi_R \\ \psi_L \end{bmatrix}$$

$$\psi_R = \Vdash \underbrace{\psi_{d_R}} \psi_{e_R} \psi_{u_R} = \begin{bmatrix} \psi_d ⧗ \begin{bmatrix} d_R \\ e_R \end{bmatrix} \\ \psi_u ⧗ u_R \end{bmatrix}$$

$$\psi_L = \begin{bmatrix} \psi_{d_L} \\ \psi_{e_L} \\ \psi_{u_L} \\ \psi_{\nu_L} \end{bmatrix} = \Psi ⧗ \begin{bmatrix} q_L \\ \ell_L \end{bmatrix}$$

$$\tilde{\psi}_3 = \begin{bmatrix} \psi_{\bar{d}_R} & \psi_{\bar{e}_R} & \psi_{\bar{u}_R} & \psi_{\bar{d}_L} & \psi_{\bar{e}_L} & \psi_{\bar{u}_L} & \psi_{\bar{\nu}_L} \end{bmatrix} = \begin{bmatrix} \bar{\psi}_R \\ \bar{\psi}_L \end{bmatrix}$$

$$\bar{\psi}_R = \begin{bmatrix} \psi_{\bar{d}_R} \\ \psi_{\bar{e}_R} \\ \psi_{\bar{u}_R} \end{bmatrix}$$

$$\bar{\psi}_L = \begin{bmatrix} \psi_{\bar{d}_L} \\ \psi_{\bar{e}_L} \\ \psi_{\bar{u}_L} \\ \psi_{\bar{\nu}_L} \end{bmatrix}$$

$$Q_3^{\mathbb{R}} = \begin{array}{c|c} Q_3 & 0 \\ \hline 0 & -Q_3 \end{array}$$

$$3Q_3 = \begin{array}{c|c} \begin{array}{c|c|c} -1 & 0 & 0 \\ \hline 0 & -3 & 0 \\ \hline 0 & 0 & 2 \end{array} & 0 \\ \hline 0 & \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} d_R \\ e_R \\ u_R \\ d_L \\ e_L \\ u_L \\ \nu_L \end{bmatrix} = \begin{array}{c} d_R \\ \hline e_R \\ \hline u_R \\ \hline d_L \\ \hline e_L \\ \hline u_L \\ \hline \nu_L \end{array}$$

$$I_3^{\mathbb{R}} = \begin{array}{c|c} I_3 & 0 \\ \hline 0 & \widetilde{I}_3 \end{array}$$

$$I_3 = \begin{array}{c|c|c} I \boxtimes \begin{array}{c|c} x/3 & 0 \\ \hline 0 & y \end{array} & 0 & 0 \\ \hline 0 & I \boxtimes x/3 & 0 \\ \hline 0 & 0 & I \boxtimes \begin{array}{c|c} x/3 & 0 \\ \hline 0 & y \end{array} \end{array}$$

$$\widetilde{I}_3 = \begin{array}{c|c|c} I \boxtimes \begin{array}{c|c} \widetilde{x}/3 & 0 \\ \hline 0 & \widetilde{y} \end{array} & 0 & 0 \\ \hline 0 & I \boxtimes \widetilde{x} & 0 \\ \hline 0 & 0 & I \boxtimes \begin{array}{c|c} \widetilde{x}/3 & 0 \\ \hline 0 & \widetilde{y} \end{array} \end{array}$$

$$3 Q_3 = \begin{array}{c|c} \begin{array}{c|c|c} I \boxtimes \begin{array}{c|c} x/3 & 0 \\ \hline 0 & y \end{array} & 0 & 0 \\ \hline 0 & I \boxtimes x/3 & 0 \\ \hline 0 & 0 & I \boxtimes \begin{array}{c|c} x/3 & 0 \\ \hline 0 & y \end{array} \end{array} & 0 \\ \hline 0 & \begin{array}{c|c|c} I \boxtimes \begin{array}{c|c} \widetilde{x}/3 & 0 \\ \hline 0 & \widetilde{y} \end{array} & 0 & 0 \\ \hline 0 & I \boxtimes \widetilde{x} & 0 \\ \hline 0 & 0 & I \boxtimes \begin{array}{c|c} \widetilde{x}/3 & 0 \\ \hline 0 & \widetilde{y} \end{array} \end{array} \end{array}$$

$$\Gamma_3^{\mathbb{R}} = \begin{bmatrix} \Gamma \boxtimes 1_R & 0 & 0 & 0 \\ 0 & -\Gamma \boxtimes 1_L & 0 & 0 \\ 0 & 0 & -\varkappa \Gamma \boxtimes \bar{1}_R & 0 \\ 0 & 0 & 0 & \varkappa \Gamma \boxtimes \bar{1}_L \end{bmatrix} = \begin{array}{c|c} \Gamma_3 & 0 \\ \hline 0 & -\varkappa \Gamma_3 \end{array} = \begin{array}{c|c} \Gamma_3 & 0 \\ \hline 0 & \varkappa J \Gamma_3 J \end{array}$$

$$\Gamma_3 = \begin{array}{c|c|c} \Gamma \boxtimes \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} & 0 & 0 \\ \hline 0 & \Gamma \boxtimes 1 & 0 \\ \hline 0 & 0 & -\Gamma \boxtimes \begin{array}{c|c} 1 & 0 \\ \hline 0 & 1 \end{array} \end{array}$$

$$\widetilde{I} = \gamma^0\,\gamma^{2-}$$

$$J_3^{\mathbb{R}} = \begin{bmatrix} 0 & 0 & \widetilde{I}\boxtimes\bar{1}_{\bar{R}} & 0 \\ 0 & 0 & 0 & \widetilde{I}\boxtimes\bar{1}_{\bar{L}} \\ -\varkappa\widetilde{I}\boxtimes\bar{1}_R & 0 & 0 & 0 \\ 0 & -\varkappa\widetilde{I}\boxtimes\bar{1}_L & 0 & 0 \end{bmatrix} = \begin{array}{c|c} 0 & J_3 \\ \hline -\varkappa J_3 & 0 \end{array}$$

$$J_3 = \begin{array}{c|c|c} \widetilde{I}\boxtimes\begin{array}{c|c} 1 & 0 \\ \hline 0 & \bar{1} \end{array} & 0 & 0 \\ \hline 0 & \widetilde{I}\boxtimes\bar{1} & 0 \\ \hline 0 & 0 & \widetilde{I}\boxtimes\begin{array}{c|c} 1 & 0 \\ \hline 0 & \bar{1} \end{array} \end{array}$$

$$\left(J_3^{\mathbb{R}}\right)^2 = \varkappa\;\mathrm{id}$$

$$\varkappa = \begin{cases} -1 & p = d/2 = 2 + + + + \\ 1 & p = 3 + - - - \end{cases}$$

$$\begin{array}{c|c} 0 & J_3 \\ \hline -\varkappa J_3 & 0 \end{array}\;\begin{array}{c|c} 0 & J_3 \\ \hline -\varkappa J_3 & 0 \end{array} = \begin{array}{c|c} -\varkappa J_3^2 & 0 \\ \hline 0 & -\varkappa J_3^2 \end{array} = \begin{array}{c|c} \varkappa & 0 \\ \hline 0 & \varkappa \end{array} \Leftarrow J_3^2 = -I$$

$$\Gamma_3^{\mathbb{R}}\,J_3^{\mathbb{R}} = -\varkappa\,J_3^{\mathbb{R}}\,\Gamma_3^{\mathbb{R}} \Leftarrow \Gamma^{\sim} = {}^{\sim}\Gamma$$

$$\Gamma_3^{\mathbb{R}}\,J_3^{\mathbb{R}} = \begin{bmatrix} \Gamma\boxtimes 1_R & 0 & 0 & 0 \\ 0 & -\Gamma\boxtimes 1_L & 0 & 0 \\ 0 & 0 & -\varkappa\Gamma\boxtimes\bar{1}_R & 0 \\ 0 & 0 & 0 & \varkappa\Gamma\boxtimes\bar{1}_L \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & \widetilde{I}\boxtimes\bar{1}_{\bar{R}} & 0 \\ 0 & 0 & 0 & \widetilde{I}\boxtimes\bar{1}_{\bar{L}} \\ -\varkappa\widetilde{I}\boxtimes\bar{1}_R & 0 & 0 & 0 \\ 0 & -\varkappa\widetilde{I}\boxtimes\bar{1}_L & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & 0 & \widetilde{\Gamma}\boxtimes\bar{1}_{\bar{R}} & 0 \\ 0 & 0 & 0 & -\widetilde{\Gamma}\boxtimes\bar{1}_{\bar{L}} \\ \widetilde{\Gamma}\boxtimes\bar{1}_R & 0 & 0 & 0 \\ 0 & -\widetilde{\Gamma}_{(8)}\boxtimes_3 L & 0 & 0 \end{bmatrix}$$

$$J_3^{\mathbb{R}}\,\Gamma_3^{\mathbb{R}} = \begin{bmatrix} 0 & 0 & \widetilde{I}\,⧗\,\bar{1}_{\bar{R}} & 0 \\ 0 & 0 & 0 & \widetilde{I}\,⧗\,\bar{1}_{\bar{L}} \\ -\varkappa\widetilde{I}\,⧗\,\bar{1}_R & 0 & 0 & 0 \\ 0 & -\varkappa\widetilde{I}\,⧗\,\bar{1}_L & 0 & 0 \end{bmatrix} \begin{bmatrix} \Gamma\,⧗\,1_R & 0 & 0 & 0 \\ 0 & -\Gamma\,⧗\,1_L & 0 & 0 \\ 0 & 0 & -\varkappa\Gamma\,⧗\,\bar{1}_R & 0 \\ 0 & 0 & 0 & \varkappa\Gamma\,⧗\,\bar{1}_L \end{bmatrix} =$$

$$\varkappa \begin{bmatrix} 0 & 0 & -\widetilde{\Gamma}\,⧗\,\bar{1}_{\bar{R}} & 0 \\ 0 & 0 & 0 & \widetilde{\Gamma}\,⧗\,\bar{1}_{\bar{L}} \\ -\widetilde{\Gamma}\,⧗\,\bar{1}_R & 0 & 0 & 0 \\ 0 & \widetilde{\Gamma}\,⧗\,\bar{1}_L & 0 & 0 \end{bmatrix}$$

$$\Gamma_3^{\mathbb{R}}\,J_3^{\mathbb{R}} = \begin{array}{c|c} \Gamma_3 & 0 \\ \hline 0 & -\varkappa\Gamma_3 \end{array}\;\begin{array}{c|c} 0 & J_3 \\ \hline -\varkappa J_3 & 0 \end{array} = \begin{array}{c|c} 0 & \Gamma_3 J_3 \\ \hline \Gamma_3 J_3 & 0 \end{array} \quad J_3^{\mathbb{R}}\,\Gamma_3^{\mathbb{R}} = \begin{array}{c|c} 0 & J_3 \\ \hline -\varkappa J_3 & 0 \end{array}\;\begin{array}{c|c} \Gamma_3 & 0 \\ \hline 0 & -\varkappa\Gamma_3 \end{array}$$

$$= \begin{array}{c|c} 0 & -\varkappa J_3\Gamma_3 \\ \hline -\varkappa J_3\Gamma_3 & 0 \end{array} = -\varkappa\,\Gamma_3^{\mathbb{R}}\,J_3^{\mathbb{R}} \Leftarrow J_3\,\Gamma_3 = \Gamma_3\,J_3$$