

$$\overline{\mathfrak{b}_\mu \partial - \mathfrak{A}_\mu}^\nu = \mathfrak{b}_\mu^\nu - \mathfrak{b}_\mu^\alpha \mathfrak{A}_\alpha^\nu$$

$$\mathfrak{A}_i \overline{\mu \partial - \mathfrak{A}_\mu} = \mathfrak{A}_i^j \mathfrak{A}_j$$

$$2 \mathfrak{A}_c^\mu \mathfrak{A}_\mu^a \eta_{ae} = \underbrace{\mathfrak{A}_c^\mu \mathfrak{A}_e^\nu - \mathfrak{A}_e^\mu \mathfrak{A}_c^\nu}_{\mu} \mathfrak{A}_\mu^f \eta_{fb} - \eta_{cf} \underbrace{\mathfrak{A}_e^\mu \mathfrak{A}_b^\nu - \mathfrak{A}_b^\mu \mathfrak{A}_e^\nu}_{\mu} \mathfrak{A}_\mu^f + \underbrace{\mathfrak{A}_b^\mu \mathfrak{A}_c^\nu - \mathfrak{A}_c^\mu \mathfrak{A}_b^\nu}_{\mu} \mathfrak{A}_\mu^a \eta_{ae}$$

$$\Omega_{mn}^k = \mathfrak{A}_n^\nu \mathfrak{A}_\nu^\mu \mathfrak{A}_\mu^k - \mathfrak{A}_m^\mu \mathfrak{A}_\nu^\nu \mathfrak{A}_\mu^k = \underbrace{\mathfrak{A}_m^\mu \mathfrak{A}_n^\nu - \mathfrak{A}_n^\mu \mathfrak{A}_m^\nu}_{\mu} \mathfrak{A}_\mu^k$$

$$\Omega_{mn}^k = -\Omega_{nm}^k$$

$$\mathfrak{A}_\mu \partial \mathfrak{A}_\nu^m - \mathfrak{A}_\nu^\alpha \mathfrak{A}_\alpha^m = -\omega_{\mu\nu}^m \mathfrak{A}_\nu^m$$

$$\mathfrak{A}_\lambda^i \eta_{kj} + \eta_{ik} \mathfrak{A}_j^k = 0$$

$$\mathfrak{b}_\lambda \mathfrak{A}_\lambda = \mathfrak{b}_\lambda \overline{\partial - \mathfrak{A}_\lambda} \mathfrak{A}_\lambda + \mathfrak{b}_\lambda \mathfrak{A}_\lambda \overline{\partial - \mathfrak{A}_\lambda} \Rightarrow$$

$$\begin{aligned} 0 &= \overbrace{\mathfrak{A}_i^\mu \mathfrak{A}_\mu^\nu}^{\eta_{ij}} = \mathfrak{A}_i \mathfrak{A}_j = \underbrace{\mathfrak{A}_i \overline{\partial - \mathfrak{A}_\lambda}}_{\lambda} \mathfrak{A}_j + \mathfrak{A}_i \mathfrak{A}_j \overline{\partial - \mathfrak{A}_\lambda} = \overbrace{\mathfrak{A}_i^k \mathfrak{A}_k} \mathfrak{A}_j + \mathfrak{A}_i \mathfrak{A}_j \overbrace{\mathfrak{A}_\lambda^k} \\ &= \overbrace{\mathfrak{A}_\lambda^k \mathfrak{A}_k}^\mu \mathfrak{A}_\mu^\nu \mathfrak{A}_j + \mathfrak{A}_i^\mu \mathfrak{A}_\mu^\nu \overbrace{\mathfrak{A}_\lambda^k \mathfrak{A}_k}^\nu = \mathfrak{A}_\lambda^k \mathfrak{A}_k^\mu \mathfrak{A}_\mu^\nu \mathfrak{A}_j + \mathfrak{A}_i^\mu \mathfrak{A}_\mu^\nu \mathfrak{A}_k^\lambda \mathfrak{A}_j = \mathfrak{A}_\lambda^k \eta_{kj} + \eta_{ik} \mathfrak{A}_j^k \end{aligned}$$

$$D_\mu \psi = \mathfrak{A}_\mu \partial \psi - \frac{1}{4} \mathfrak{A}_\mu^{mn} \Gamma_{mn} \psi = \mathfrak{A}_\mu \partial \psi - \frac{1}{4} \mathfrak{A}_\mu^j \eta_{jk} \mathfrak{A}_i^i \times \mathfrak{A}_k \psi$$

$$\Gamma_{mn} = \frac{1}{2} \Gamma_m \times \Gamma_n$$

$$\mathfrak{b}_b \mathfrak{g}_b^d \mathfrak{A}_b^o = \mathfrak{b} + \overline{d^i \mathfrak{b}_b g}$$

$$\mathfrak{b}_b \mathfrak{g}_b^d \mathfrak{A}_b^o = \mathfrak{b} + \mathfrak{b}_k \mathfrak{b}_\ell \overline{\mathfrak{b}_b \mathfrak{A}_b^k} \times \overline{\mathfrak{b}_b \mathfrak{A}_b^\ell}$$

$$\mathfrak{b}_k \mathfrak{b}_\ell + \mathfrak{b}_\ell \mathfrak{b}_k = 0 \Rightarrow$$

$$\begin{aligned} \overline{\mathfrak{b}_k} \overline{d^i \mathfrak{b}_b g}^j &= \overline{\mathfrak{b}_k \mathfrak{A}_b^i \mathfrak{A}_b^j} = \overline{\mathfrak{b}_k \mathfrak{A}_b^i \mathfrak{A}_b^j} \mathfrak{A}_b^o = \overline{\mathfrak{b}_k \mathfrak{g}_b^d} \times \mathfrak{A}_b^j \mathfrak{A}_b^o = \overline{\mathfrak{b}_k \mathfrak{g}_b^d \mathfrak{A}_b^o} \times \overline{\mathfrak{b}_k} \\ &= \mathfrak{b} \times \overline{\mathfrak{b}_k} + \mathfrak{b}_k \mathfrak{b}_\ell \overline{\mathfrak{b}_b \mathfrak{A}_b^k} \times \overline{\mathfrak{b}_b \mathfrak{A}_b^\ell} \times \mathfrak{b}_k \mathfrak{A}_b^j = \mathfrak{b} \times \overline{\mathfrak{b}_k} + 2 \mathfrak{b}_k \mathfrak{b}_\ell \overline{\mathfrak{b}_b \mathfrak{A}_b^k} \eta^{\ell j} - \eta^{jk} \overline{\mathfrak{b}_b \mathfrak{A}_b^\ell} \end{aligned}$$

$$= 2 \underbrace{b_\ell}_{\mathbb{C}} \underbrace{a^k}_{\mathbb{C}} \eta^{jj} - 2 \underbrace{b_\ell}_{\mathbb{C}} \eta^{jj} \underbrace{a^\ell}_{\mathbb{C}} = 4 \underbrace{a^k}_{\mathbb{C}} \underbrace{b_j}_{\mathbb{C}} \eta^{jj} \Rightarrow \underbrace{b_j}_{\mathbb{C}} = \frac{1}{4} \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}} \eta$$

$$\underbrace{g^d}_{\mathbb{C} \times \mathbb{C}} \underbrace{a^\circ}_{\mathbb{C}} = \underbrace{a}_{\mathbb{C}} \times \underbrace{a}_{\mathbb{C}} + \overbrace{\underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}}}_{\mathbb{C}} - \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}} \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}}$$

$$\underbrace{a}_{\mathbb{C}} \times \underbrace{a}_{\mathbb{C}} \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}} = \overbrace{\underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}}}_{\mathbb{C}} - \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}} \underbrace{a}_{\mathbb{C}} \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}} = \overbrace{\underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}}}_{\mathbb{C}} - \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}} \underbrace{d^i}_{\mathbb{C}} \underbrace{a^j}_{\mathbb{C}}$$

$$\underbrace{g^d}_{\lambda^{\mathbb{C}}} \times \underbrace{\tilde{a}^\nu}_{\mathbb{C}} = \underbrace{\tilde{a}^\mu}_{\lambda} \underbrace{d^i}_{\lambda} \underbrace{a^j}_{\mu} \underbrace{\nu}_{\mu}$$

$$\text{LHS} = \underbrace{\lambda}_{\lambda} \underbrace{d^i}_{\lambda} \underbrace{a^j}_{\mu} \underbrace{\tilde{a}^\nu}_{\mathbb{C}} = \underbrace{\tilde{a}^\mu}_{\lambda} \underbrace{d^i}_{\lambda} \underbrace{a^j}_{\mu} \underbrace{\nu}_{\mu} = \text{RHS}$$

$$\underbrace{\tilde{a}^\lambda}_{\lambda^{\mathbb{C}}} \underbrace{g^d}_{\lambda^{\mathbb{C}}} \times \underbrace{\tilde{a}^\nu}_{\mathbb{C}} = \underbrace{\tilde{a}^\lambda}_{\lambda} \times \underbrace{\tilde{a}^\mu}_{\lambda} \underbrace{d^i}_{\lambda} \underbrace{a^j}_{\mu} \underbrace{\nu}_{\mu} \text{ id}$$

$$\underbrace{d^i}_{\lambda} \underbrace{a^j}_{\mu} \underbrace{\nu}_{\mu} = \underbrace{d^i}_{\mu} \underbrace{a^j}_{\lambda} \underbrace{\nu}_{\lambda}$$

$$\Rightarrow \text{LHS} = \underbrace{\tilde{a}^\lambda}_{\lambda} \underbrace{\tilde{a}^\mu}_{\lambda} \underbrace{d^i}_{\lambda} \underbrace{a^j}_{\mu} \underbrace{\nu}_{\mu} = \frac{\tilde{a}^\lambda \tilde{a}^\mu + \tilde{a}^\mu \tilde{a}^\lambda}{2} \underbrace{d^i}_{\lambda} \underbrace{a^j}_{\mu} \underbrace{\nu}_{\mu} = \text{RHS}$$