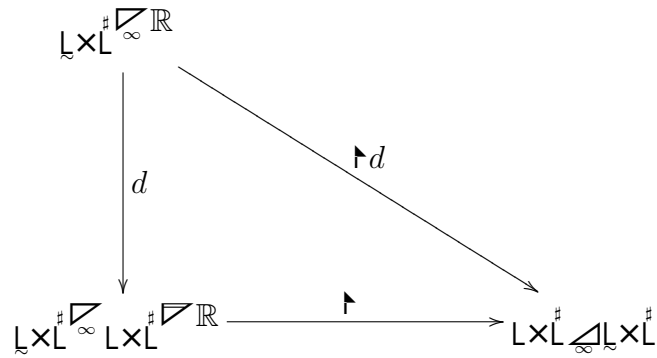


$\mathbb{L} \times \mathbb{L}^\# \in \mathbb{R}_\infty^\#$ symplectic



$$\mathbb{L} \times \mathbb{L}^\#_{h:\mathcal{V}} = \mathbb{L} \times \mathbb{L}^\# \ni L:\mathbb{1} \mapsto \underline{L:\mathbb{1}}_{h:\mathcal{V}} = L\mathbb{1} + \mathbb{1}\mathbb{1} \in \mathbb{R} \Rightarrow \overbrace{\mathbb{1}\mathbb{1}}^{h:\mathcal{V}} = \underline{-\mathbb{1}:\mathbb{1}} \in \mathbb{L} \times \mathbb{L}^\#_{h:\mathcal{V}}$$

$$\begin{bmatrix} \mathbb{1}\mathbb{1} \\ \mathbb{1} \end{bmatrix}_{\mathcal{V}} = \mathbb{1}\mathbb{1} \Rightarrow \begin{bmatrix} \mathbb{1}\mathbb{1} \\ L:\mathbb{1} \end{bmatrix}_{h:\mathcal{V}} = \underline{L:\mathbb{1}}_{h:\mathcal{V}} = L\mathbb{1} + \mathbb{1}\mathbb{1} = \begin{bmatrix} -\mathbb{1}:\mathbb{1} \\ L:\mathbb{1} \end{bmatrix}_{h:\mathcal{V}}$$

$$\mathbb{L} \times \mathbb{L}^\# \ni h:\mathcal{V} \cap_{h:\mathcal{V}} \in \mathbb{R} \Rightarrow \overbrace{d\mathbb{1}\mathbb{1}}^{h:\mathcal{V}} = \begin{bmatrix} -\frac{\partial \mathbb{1}}{\partial \mathcal{V}} & \frac{\partial \mathbb{1}}{\partial h} \end{bmatrix} \in \mathbb{L} \times \mathbb{L}^\# = \mathbb{L}^\#_{\mathcal{V}} \times \mathbb{L}^\#_h$$

$$\mathbb{L} \times \mathbb{L}^\#_{h:\mathcal{V}} = \mathbb{L} \times \mathbb{L}^\# \ni L:\mathbb{1} \mapsto \underline{L:\mathbb{1}}_{h:\mathcal{V}} d\mathbb{1} = L \frac{\partial \mathbb{1}}{\partial h} + \frac{\partial \mathbb{1}}{\partial \mathcal{V}} \mathbb{1}$$

$$\mathbb{1} \times \mathbb{1}_{h:\mathcal{V}} = -\frac{\partial \mathbb{1}}{\partial \mathcal{V}} \frac{\partial \mathbb{1}}{\partial h} + \frac{\partial \mathbb{1}}{\partial \mathcal{V}} \frac{\partial \mathbb{1}}{\partial h}$$

$$\text{LHS} = \begin{bmatrix} d\mathbb{1}\mathbb{1} \\ d\mathbb{1}\mathbb{1} \end{bmatrix}_{h:\mathcal{V}} = \begin{bmatrix} -\frac{\partial \mathbb{1}}{\partial \mathcal{V}} \frac{\partial \mathbb{1}}{\partial h} \\ \frac{\partial \mathbb{1}}{\partial \mathcal{V}} \frac{\partial \mathbb{1}}{\partial h} \\ -\frac{\partial \mathbb{1}}{\partial \mathcal{V}} \frac{\partial \mathbb{1}}{\partial h} \end{bmatrix}_{h:\mathcal{V}} = \text{RHS}$$

$$\underbrace{(\downarrow)}_{\uparrow} \uparrow \downarrow = \downarrow$$

$$\uparrow \uparrow \underbrace{(\downarrow)}_{\uparrow} = \uparrow$$

$$\uparrow \underbrace{(\downarrow \uparrow)}_{\uparrow} \downarrow = \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow = \uparrow \downarrow$$

$$\begin{bmatrix} \uparrow \uparrow \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow = \uparrow \underbrace{(\uparrow \downarrow)}_{\uparrow} = \begin{bmatrix} \uparrow \\ \uparrow \end{bmatrix} \uparrow$$

$$\downarrow \times \uparrow = \underbrace{(\downarrow \uparrow)}_{\uparrow} \times \downarrow$$

$$\underbrace{(\downarrow \times \uparrow)}_{\uparrow} = \downarrow \times \uparrow$$

$$\downarrow \times \uparrow = \underbrace{(\downarrow)}_{\uparrow} \times \uparrow - \underbrace{(\downarrow)}_{\uparrow} \times \downarrow + d \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow$$

$$\begin{aligned} 0 &= \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \underbrace{(\downarrow)}_{\uparrow} = \underbrace{(\downarrow)}_{\uparrow} \times \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow - \underbrace{(\downarrow)}_{\uparrow} \times \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow + d \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow - \begin{bmatrix} \downarrow \uparrow \times \uparrow \\ \uparrow \end{bmatrix} \downarrow + \begin{bmatrix} \downarrow \uparrow \times \uparrow \\ \uparrow \end{bmatrix} \downarrow - \begin{bmatrix} \downarrow \uparrow \times \uparrow \\ \uparrow \end{bmatrix} \downarrow \\ &= \underbrace{(\downarrow)}_{\uparrow} \times \underbrace{(\uparrow \downarrow)}_{\uparrow} - \underbrace{(\downarrow)}_{\uparrow} \times \underbrace{(\uparrow \downarrow)}_{\uparrow} + d \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow - \uparrow \underbrace{(\downarrow \times \uparrow)}_{\uparrow} - \underbrace{(\downarrow \times \uparrow \uparrow)}_{\uparrow} + \underbrace{(\downarrow \times \uparrow \uparrow)}_{\uparrow} \\ &= \uparrow \underbrace{(\downarrow \times \uparrow - \downarrow \times \uparrow + d \begin{bmatrix} \downarrow \uparrow \\ \uparrow \end{bmatrix} \downarrow - \downarrow \times \uparrow)}_{\uparrow} \leftarrow \uparrow \underbrace{(\downarrow \times \uparrow)}_{\uparrow} = \underbrace{(\downarrow)}_{\uparrow} \times \underbrace{(\uparrow \downarrow)}_{\uparrow} + \uparrow \times \underbrace{(\downarrow \uparrow)}_{\uparrow} \end{aligned}$$

$$\downarrow \times \uparrow = \begin{bmatrix} \downarrow \uparrow \\ \downarrow \uparrow \end{bmatrix} \downarrow = \underbrace{(\downarrow \uparrow)}_{\uparrow} \times \uparrow$$

$$d \underbrace{(\downarrow \times \uparrow)}_{\uparrow} = \underbrace{(\downarrow)}_{\uparrow} \times \underbrace{(\downarrow \uparrow)}_{\uparrow}$$

$$\underbrace{(\downarrow \times \uparrow \uparrow)}_{\uparrow} = \underbrace{(\downarrow \uparrow)}_{\uparrow} \times \underbrace{(\downarrow \uparrow)}_{\uparrow}$$

$$\begin{aligned} \underbrace{(\downarrow)}_{\uparrow} \times \underbrace{(\downarrow \uparrow)}_{\uparrow} &= \underbrace{(\downarrow \uparrow)}_{\uparrow} \times \underbrace{(\downarrow \uparrow)}_{\uparrow} - \underbrace{(\downarrow \uparrow)}_{\uparrow} \times \underbrace{(\downarrow)}_{\uparrow} + d \begin{bmatrix} \downarrow \uparrow \\ \downarrow \uparrow \end{bmatrix} \downarrow \\ &= d \underbrace{(\downarrow \uparrow)}_{\uparrow} \times \uparrow - d \underbrace{(\downarrow \uparrow)}_{\uparrow} \times \downarrow + d \uparrow \times \downarrow = d \underbrace{(\downarrow \times \uparrow - \downarrow \times \downarrow + \uparrow \times \downarrow)}_{\uparrow} \end{aligned}$$