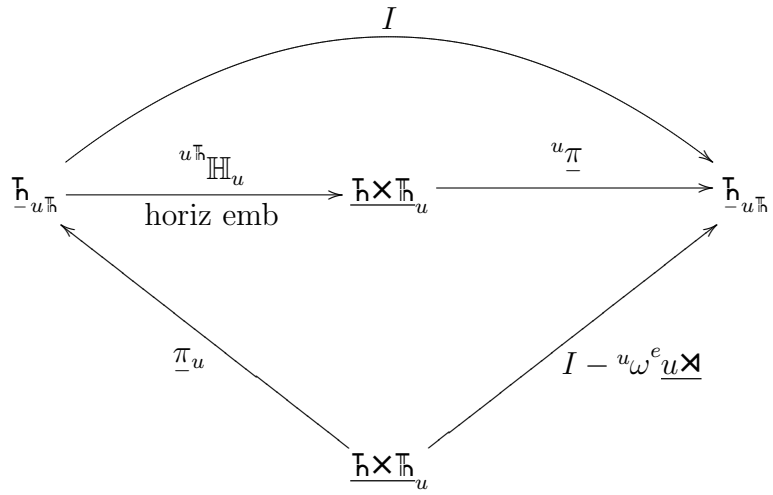
$$\begin{array}{ccc} \underline{\mathbb{h}\times\mathbb{h}}_u & \sqsubset & \underline{\mathbb{h}\times\mathbb{h}} \\ \downarrow {}^u\underline{\pi} & & \downarrow \underline{\pi} \\ \underline{\mathbb{h}}_{u\mathbb{h}} & \sqsubset & \underline{\mathbb{h}} \end{array}$$

$$\text{vert } \underline{\mathbb{h}}_u = \ker {}^u\underline{\pi} = \begin{cases} A^e\underline{u\bowtie} \\ A \in \underline{\mathbb{h}}_e \end{cases} \quad \sqsubset \underline{\mathbb{h}\times\mathbb{h}}_u$$

$$\begin{array}{ccccc} \mathbb{h} & \xrightarrow{u\bowtie} & \mathbb{h}\times\mathbb{h} & \xrightarrow{\pi} & \mathbb{h} \\ & \searrow & \text{cst} & \nearrow & \end{array}$$

$$\begin{array}{ccccc} \underline{\mathbb{h}}_e & \xrightarrow{{}^e\underline{u\bowtie}} & \underline{\mathbb{h}\times\mathbb{h}}_u & \xrightarrow{{}^u\underline{\pi}} & \underline{\mathbb{h}}_{u\mathbb{h}} \\ & \searrow & 0 & \nearrow & \end{array}$$



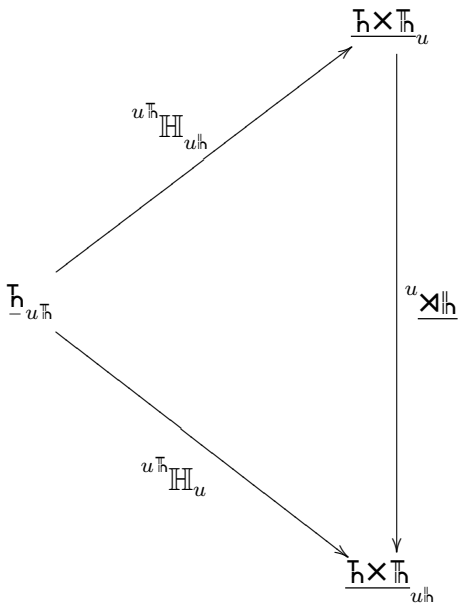
$$X\,{}^{u}\underline{\pi} = 0 \Longrightarrow X = A\,{}^{e}\underline{u\bowtie} \Longrightarrow X\,{}^{u}\omega = A \Longrightarrow X\underbrace{I - {}^{u}\omega^{e}\underline{u\bowtie}} = X - \underbrace{X{}^{u}\omega}\,{}^{e}\underline{u\bowtie} = A\,{}^{e}\underline{u\bowtie} - A\,{}^{e}\underline{u\bowtie} = 0$$

$$\Longrightarrow {}^{u\mathbb{h}}\mathbb{H}_u \text{ well-def}$$

$${}^{u\mathbb{h}}\mathbb{H}_u\,{}^{u}\underline{\pi} = I$$

$$\text{horiz } \underline{\mathbb{h}}_u = \underline{\mathbb{h}}_{u\mathbb{h}}\,{}^{u\mathbb{h}}\mathbb{H}_u \sqsubset \underline{\mathbb{h}\times\mathbb{h}}_u$$

$$\underline{\mathbb{h}\times\mathbb{h}}_u = \underline{\mathbb{h}}_u \oplus \underline{\mathbb{h}}_u$$


$\underline{\mathbb{h}\times\mathbb{h}}_{u}$
$u^{\mathbb{h}}\mathbb{H}_{u\mathbb{h}}$
$\underline{\mathbb{h}}_{u\mathbb{h}}$
$u\underline{\bowtie\mathbb{h}}$
$u^{\mathbb{h}}\mathbb{H}_{u}$
$\underline{\mathbb{h}\times\mathbb{h}}_{u\mathbb{h}}$