

$$\mathbf{J} \times \mathbf{J} = \frac{\partial \mathbf{J}}{\partial_i \mathbf{q}} \frac{\partial \mathbf{J}}{\partial \mathbf{L}^i} - \frac{\partial \mathbf{J}}{\partial_i \mathbf{p}} \frac{\partial \mathbf{J}}{\partial \mathbf{L}^i}$$

$$\mathfrak{h} \times_n \mathbb{R} \xrightarrow[H]{\mathbf{J}} \mathbb{R}$$

$$\dot{\mathbf{L}}^i = H \times \mathbf{L}^i = \frac{\partial H}{\partial_i \mathbf{q}}$$

$${}_i \dot{\mathbf{p}} = H \times {}_i \mathbf{p} = -\frac{\partial H}{\partial \mathbf{L}^i}$$

$$H \times \mathbf{L}^i = \frac{\partial H}{\partial_j \mathbf{q}} \frac{\partial \mathbf{L}^i}{\partial \mathbf{L}^j} - \frac{\partial \mathbf{L}^i}{\partial_j \mathbf{q}} \frac{\partial H}{\partial \mathbf{L}^j} = \frac{\partial H}{\partial_i \mathbf{q}}$$

$$H \times {}_i \mathbf{p} = \frac{\partial H}{\partial_j \mathbf{q}} \frac{\partial {}_i \mathbf{p}}{\partial \mathbf{L}^j} - \frac{\partial {}_i \mathbf{p}}{\partial_j \mathbf{q}} \frac{\partial H}{\partial \mathbf{L}^j} = -\frac{\partial H}{\partial \mathbf{L}^i}$$

$$\partial_t \mathbf{J} = H \times \mathbf{J}$$

$$\text{LHS} = \underbrace{\partial_t \mathbf{L}^i}_{\dot{\mathbf{L}}^i} \frac{\partial \mathbf{J}}{\partial \mathbf{L}^i} + \underbrace{\partial_t \mathbf{p}}_{{}_i \dot{\mathbf{p}}} \frac{\partial \mathbf{J}}{\partial_i \mathbf{p}} = \frac{\partial H}{\partial_i \mathbf{q}} \frac{\partial \mathbf{J}}{\partial \mathbf{L}^i} - \frac{\partial H}{\partial \mathbf{L}^i} \frac{\partial \mathbf{J}}{\partial_i \mathbf{p}} = \text{RHS}$$