$$\mathbb{Z} \overset{\times}{\ulcorner} \mathbb{Z}m = \frac{\mathbb{Z}m+n}{m \curlywedge n = 1}$$

$$\subset : \quad \mathbb{Z}m+1 = \underbrace{\mathbb{Z}m+b}\,\underbrace{\mathbb{Z}m+n} = \mathbb{Z}m+bn \Rightarrow 1-bn \in \mathbb{Z}m \Rightarrow 1-bn = am \Rightarrow 1 = am+bn$$

$$\supset : \quad m \curlywedge n = 1 \Rightarrow 1 = am+bn \Rightarrow \mathbb{Z}m+1 = \underbrace{\mathbb{Z}m+a}\,\underbrace{\mathbb{Z}m+m}_{=\mathbb{Z}m+0} + \underbrace{\mathbb{Z}m+b}\,\underbrace{\mathbb{Z}m+n} = \underbrace{\mathbb{Z}m+b}\,\underbrace{\mathbb{Z}m+n}$$

$$\Rightarrow \mathbb{Z}m+n \in \mathbb{Z} \overset{\times}{\ulcorner} \mathbb{Z}m$$

$$\mathbb{Z} \overset{\times}{\ulcorner} \mathbb{Z}m \xrightarrow[\text{hom}]{\chi} \mathbb{C}^{\times}$$

$$p \nmid m \Rightarrow {}^{s}_{p}\mathbb{Q}_{\chi}^{-1} = 1 - \frac{\chi\,(p+\mathbb{Z}m)}{p^{s}}$$

$${}^{s}_{8}\mathbb{Q}_{\chi} = \prod_{p \nmid m} {}^{s}_{p}\mathbb{Q}_{\chi} = \prod_{p \nmid m} \left(1 - \frac{\chi\,(p+\mathbb{Z}m)}{p^{s}}\right)^{-1} = \sum_{m \curlywedge n = 1} \frac{\chi\,(n+\mathbb{Z}m)}{n^{s}}$$

$${}^{s}_{\infty}\mathbb{Q}_{\chi} = \frac{q^{s/2}\Gamma_{s/2}}{\pi^{s/2}}$$

$${}^{s}\mathbb{Q}_{\chi} = {}^{s}_{8}\mathbb{Q}_{\chi}\,{}^{s}_{\infty}\mathbb{Q}_{\chi}$$

$${}^{1/2+s}\mathbb{Q}_{\chi} = \varepsilon_{\chi}\;{}^{1/2-s}\mathbb{Q}_{\bar{\chi}}$$

$${}^{s}_{8}\mathbb{Q}\;{}^{s}_{8}\mathbb{Q}_{\chi} \underset{\text{Gauss}}{=} {}^{s}_{8}\mathbb{Q}^{\sqrt{\chi - 1m}}$$

$$\chi_{N}$$

$$\overbrace{c\tau+d}^{-1}\underbrace{a\tau+b}\,\urcorner = \chi_{N}\,(d)\,\overbrace{c\tau+d}^{k}\,{}^{\tau}\urcorner$$

$$\begin{array}{c|c} a & b \\ \hline c & d \end{array} \in \begin{array}{c|c} \mathbb{Z}^{\complement} & \mathbb{Z} \\ \hline \mathbb{Z}N & \mathbb{Z} \end{array}$$

$${}^{2}\bar{\mathbb{Q}}_{2}^{\complement} \ltimes {}^{2}\mathbb{Q}_{2}^{\complement} \neg\, {}^{2}\bar{\mathbb{Q}}_{2}^{\complement}\, {}_{\omega}\triangle^{2}\,\mathbb{C}$$