$$\hbar \in \mathbb{K} \triangleright_0^{\omega}$$

$$\hbar \underset{\mathrm{ex}}{\subseteq} \hbar \Leftrightarrow$$

$$\hbar \vDash \overbrace{\hbar_{\neg} \hbar}_{\triangleright_0} \mathbb{K}$$

$$\hbar_i \underset{\mathrm{ex}}{\subseteq} \hbar \Rightarrow \bigcap_i \hbar_i \underset{\mathrm{ex}}{\subseteq} \hbar$$

$$\dot{\hbar} \underset{\mathrm{ex}}{\subseteq} \hbar \Rightarrow \hbar \cup \acute{\hbar} \underset{\mathrm{ex}}{\subseteq} \hbar$$

$$\overline{\hbar} = \hbar \vDash \overbrace{\hbar_{\neg} \hbar}_{\triangleright_0^{\omega}} \mathbb{K}$$

$$A_0 \subset \hbar \supset A_1$$

$$A_0 \cap A_1 = \varnothing \Rightarrow \begin{cases} \bigvee \gamma \in {}^{\hbar}\triangleright_0 \mathbb{I} \\ {}_{A_j} \gamma = j \end{cases}$$

$$\overset{\mathbb{Q} \cap {}_0\overleftrightarrow{\mathbb{R}}_1}{\bigwedge_r} \; \bigvee_{U_r \subset ℏ} A_0 \subset U_r \Subset ℏ \llcorner A_1$$

$$r < s \implies U_r \Subset U_s$$

$$r_0 = 0 : r_1 = 1 \implies \bigvee A_0 \subset U_0 \Subset U_1 \Subset ℏ \llcorner A_1$$

$$\mathbb{Q} \cap {}_0\overleftrightarrow{\mathbb{R}}_1 = \left\{ r_2 : r_3 \cdots \right\}$$

$$\text{chosen } U_{r_0} \cdots U_{r_{n-1}} \implies r_n \notin \frac{r_k}{k \in n} \implies \bigvee_{i:j \in n} \begin{cases} r_i = \overset{< r_n}{\bigvee} r_k \\ r_j = \underset{> r_n}{\bigwedge} r_k \end{cases}$$

$$\implies r_i < r_j \implies U_{r_i} \Subset U_{r_j} \implies \bigvee U_{r_i} \Subset U_{r_n} \Subset U_{r_j}$$

$$\daleth_r = r \chi_{ℏ \llcorner \hat{U}_r} \; \underline{\text{stet}} \implies ℏ \xrightarrow[\underline{\text{stet}}]{\underline{\daleth} = \bigvee_r \daleth_r} {}_{-1}\overleftrightarrow{\mathbb{R}}_1$$

$$\daleth^s = s \chi_{U_s} + \chi_{ℏ \llcorner U_s} = s + (1-s)\chi_{ℏ \llcorner U_s} \; \overline{\text{stet}} \implies ℏ \xrightarrow[\overline{\text{stet}}]{\overline{\daleth} = \bigwedge_s \daleth^s} {}_{-1}\overleftrightarrow{\mathbb{R}}_1$$

$$\underline{\daleth} = \overline{\daleth} = \daleth \text{ stet}$$

$$\lightning \bigvee_x^ℏ {}^x\underline{\daleth} > {}^x\overline{\daleth} \implies \bigvee_{r:s} {}^x\daleth_r > {}^x\daleth^s \implies x \in U_s \llcorner \hat{U}_r$$

$$r > s \implies U_r \supset \hat{U}_s \; \lightning$$

$$\lightning \bigvee_x^ℏ {}^x\underline{\daleth} < {}^x\overline{\daleth} \implies \bigvee_{r:s} {}^x\daleth_r \leqslant {}^x\underline{\daleth} < r < s < {}^x\overline{\daleth} \leqslant {}^x\daleth^s \implies x \in \hat{U}_r \llcorner U_s$$

$$\hat{U}_r \subset U_s \; \lightning$$

$$\Upsilon^{A_j} = j$$

$$x \in A_0 \Rightarrow \bigwedge_{r:s} x \in U_s \Rightarrow {}^x\Upsilon = \bigwedge_s {}^x\Upsilon^s = \bigwedge_s s = 0$$

$$x \in A_1 \Rightarrow \bigwedge_{r:s} x \in \hbar \llcorner \hat{U}_r \Rightarrow {}^x\Upsilon = \curlyvee_r {}^x\Upsilon_r = \curlyvee_r r = 1$$