$$\mathbb{Q} \qquad \sqsubset_{\text{fin}} \qquad Q \text{ number field}$$

$$\mathbb{Z} \qquad \sqsubset_{\text{fin}} \qquad Z \text{ number ring}$$

$${}^{s}_{\mathfrak{p}}Q_{\chi}^{-1} = 1 - \frac{\chi_{\mathfrak{p}}}{(\sharp\mathfrak{p})^{s}}$$

$${}^{s}_{8}Q_{\chi} = \prod_{\mathfrak{p} \triangleleft Z} {}^{s}_{\mathfrak{p}}Q_{\chi} = \prod_{\mathfrak{p} \triangleleft Z} \frac{1}{1 - \chi_{\mathfrak{p}}/(\sharp\mathfrak{p})^{s}} = \sum_{\mathfrak{a} \triangleleft Z} \frac{\chi_{\mathfrak{a}}}{(\sharp\mathfrak{a})^{s}}$$

$${}^{s}_{\infty}Q_{\chi} = \frac{\Gamma_{s/2}^{r_1}\Gamma_{s}^{r_2}}{\pi^{r_1 s/2}(2\pi)^{r_2 s}|D|\sharp\mathfrak{m}^{s/2}}$$

$${}^{s}Q_{\chi} = {}^{s}_{8}Q_{\chi}\,{}^{s}_{\infty}Q_{\chi} = {}^{1-s}Q_{\chi}$$

$$\begin{array}{ccccc} \mathbb{Q} & \subset & Q & \xrightarrow{p} & \mathbb{R}^{d_1} \times \mathbb{C}^{d_2} \\ \cup & & \cup & & \cup \\ \mathbb{Z} & \subset & Z & \xrightarrow{\asymp} & \mathbb{Z}^{d_1 + d_2} \end{array}$$

$$D = \det\left(\omega_i^{p}\right)$$

$$\begin{cases} N \triangleleft Z \\ M \curlywedge M = 1 \end{cases} \quad \xrightarrow[\text{hom}]{\chi} \quad \mathbb{C}^{x}$$

$${}^{s}_{\chi}\zeta_{Q} = \sum_{N \triangleleft Z} \frac{\chi(N)}{\ulcorner N \urcorner^{s}}$$

$${}^{s}_{\chi}\zeta_{Q}^{-1} = \prod_{P \triangleleft Z} \left(1 - \frac{\chi(P)}{\ulcorner Z \sqcap P \urcorner^{s}}\right)$$