$$\mathbb{C}_r^{\mathsf{C}} = \frac{\zeta = \xi + \eta \in \mathbb{C}_r^{\mathsf{C}}}{\xi \in \mathbb{R}_r \ni i\eta}$$

$$\downarrow$$

$$\mathbb{R}^{\mathsf{U}} \ulcorner \mathbb{C}_r^{\mathsf{C}} = \frac{o \cdot \zeta = o \cdot \xi + o \cdot \eta}{o = \pm \in \mathbb{R}^{\mathsf{U}}}$$

$${}^{\ell}_{=}\mathbb{C}_r^{\mathsf{C}} = \frac{\zeta = \xi + \eta \in {}^{\ell}\mathbb{C}_r^{\mathsf{C}}}{\xi \in {}^{\ell}\mathbb{R}_r \ni i\eta\colon \quad \xi\overset{*}{\eta} + \eta\overset{*}{\xi} = 0}$$

$$\downarrow$$

$${}^{\ell}\mathbb{R}_{\ell}^{\mathsf{U}} \ulcorner {}^{\ell}_{=}\mathbb{C}_r^{\mathsf{C}} = \frac{o \cdot \zeta = o \cdot \xi + o \cdot \eta}{o \in {}^{\ell}\mathbb{R}_{\ell}^{\mathsf{U}}}$$

$$\begin{bmatrix} \eta & \xi \end{bmatrix} J \begin{bmatrix} \overset{*}{\eta} \\ \overset{*}{\xi} \end{bmatrix} = \begin{bmatrix} \eta & \xi \end{bmatrix} \begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array} \begin{bmatrix} \overset{*}{\eta} \\ \overset{*}{\xi} \end{bmatrix} = \begin{bmatrix} \xi & \eta \end{bmatrix} \begin{bmatrix} \overset{*}{\eta} \\ \overset{*}{\xi} \end{bmatrix} = \xi\overset{*}{\eta} + \eta\overset{*}{\xi}$$

$$\zeta\overset{*}{\zeta} = \overset{-}{\zeta}\,\overset{t}{\zeta} \Leftrightarrow \xi\overset{*}{\eta} + \eta\overset{*}{\xi} = \begin{bmatrix} \eta & \xi \end{bmatrix} J \begin{bmatrix} \overset{*}{\eta} \\ \overset{*}{\xi} \end{bmatrix} = 0$$

$$\zeta\overset{*}{\zeta} = \underbrace{\xi + \eta}\underbrace{\overset{*}{\xi} + \overset{*}{\eta}} = \underbrace{\xi\overset{*}{\xi} + \eta\overset{*}{\eta}}_{\text{symm}} + \underbrace{\xi\overset{*}{\eta} + \eta\overset{*}{\xi}}_{\text{asym}}$$

$$\zeta = \lambda\vartheta \begin{cases} \vartheta\overset{*}{\vartheta} = 1 = \overset{-}{\vartheta}\overset{t}{\vartheta} \\ \lambda = \overset{t}{\lambda} = \overset{*}{\lambda} \end{cases}$$

$$\vartheta = \sigma + \tau \begin{cases} \sigma\overset{*}{\sigma} + \tau\overset{*}{\tau} = 1 \\ \sigma\overset{*}{\tau} + \tau\overset{*}{\sigma} = 0 \end{cases}$$

$$\vartheta\overset{*}{\vartheta} = \underbrace{\sigma + \tau}\underbrace{\overset{*}{\sigma} + \overset{*}{\tau}} = \underbrace{\sigma\overset{*}{\sigma} + \tau\overset{*}{\tau}}_{\text{symm}} + \underbrace{\sigma\overset{*}{\tau} + \tau\overset{*}{\sigma}}_{\text{asym}}$$

$$\dim_{\mathbb{R}} {}^{\ell}_{=}\mathbb{C}_r^{\mathsf{C}} = 2\ell r - \frac{\ell(\ell - 1)}{2}$$

$$\dim_{\mathbb{R}} \mathbb{R}^{\mathtt{U}} \ulcorner {}^{\ell}_{=}\mathbb{C}_r^{\mathtt{C}} = 2\ell r - \frac{\ell(\ell-1)}{2} - \frac{\ell(\ell-1)}{2} = \ell(2r+1-\ell)$$

$$\zeta_{\mathbb{C}} = \begin{bmatrix} \bar{\zeta} \\ \zeta \end{bmatrix}$$

$$\overset{*}{\textcircled{c}} = \begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array}$$

$$\zeta_{\mathbb{R}} = \overset{*}{\textcircled{c}}\, \zeta_{\mathbb{C}}$$

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{array}{c|c} 1 & 1 \\ \hline i & -i \end{array} \begin{bmatrix} \bar{\zeta} \\ \zeta \end{bmatrix}$$

$$\zeta \overset{*}{\zeta} = \lambda \overset{*}{\vartheta} \vartheta \lambda = \overset{2}{\lambda}$$

$$\vartheta = \zeta \overset{-1/2}{\lambda} = \zeta \underbrace{\overset{*}{\zeta}\zeta}_{-1/2}$$

$$\zeta_{\mathbb{R}} = \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$

$$\textcircled{c} = \begin{array}{c|c} 1 & -i \\ \hline 1 & i \end{array}$$

$$\zeta_{\mathbb{C}} = \textcircled{c}\, \zeta_{\mathbb{R}}$$

$$\begin{bmatrix} \bar{\zeta} \\ \zeta \end{bmatrix} = \begin{array}{c|c} 1 & -i \\ \hline 1 & i \end{array} \begin{bmatrix} \xi \\ \eta \end{bmatrix}$$