$$\underline{P}_w = P \times \underline{\pi}_w$$

$$P\,\pi_{g_w} = \pi_{g_w}\,P_w \Longrightarrow P\,\underline{\pi}_w = \underline{\pi}_w\,P + \underline{P}_w$$

$$\underline{\pi}_w = \begin{array}{c|c} a_w & -c_w^* \\ \hline c_w & d_w \end{array} \;\Longrightarrow\; \Omega_{v:w} = c_w^*\,c_v - c_v^*\,c_w$$

$$\underline{P}_w = P \times \underline{\pi}_w = \begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array} \times \begin{array}{c|c} a_w & -c_w^* \\ \hline c_w & d_w \end{array} = \begin{array}{c|c} 0 & c_w^* \\ \hline -c_w & 0 \end{array}$$

$$\text{LHS} = P\overbrace{\underline{P}_v \times \underline{P}_w}P = \begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array} \underbrace{\begin{array}{c|c} 0 & c_v^* \\ \hline -c_v & 0 \end{array} \times \begin{array}{c|c} 0 & c_w^* \\ \hline -c_w & 0 \end{array}} \begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array}$$

$$= \begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array} \begin{array}{c|c} c_w^* c_v - c_v^* c_w & 0 \\ \hline 0 & c_w c_v^* - c_v c_w^* \end{array} \begin{array}{c|c} 1 & 0 \\ \hline 0 & 0 \end{array} = \begin{array}{c|c} c_w^* c_v - c_v^* c_w & 0 \\ \hline 0 & 0 \end{array} = \text{RHS}$$

$$\underline{\pi}_v \times \underline{\pi}_w = \begin{array}{c|c} a_v \times a_w + \Omega_{v:w} & 0 \\ \hline 0 & d_v \times d_w + c_w c_v^* - c_v c_w^* \end{array}$$

$$\text{LHS} = \begin{array}{c|c} a_v & -c_v^* \\ \hline c_v & d_v \end{array} \times \begin{array}{c|c} a_w & -c_w^* \\ \hline c_w & d_w \end{array} = \begin{array}{c|c} a_v a_w - c_v^* c_w - a_w a_v + c_w^* c_v & -a_v c_w^* - c_v^* d_w + a_w c_v^* + c_w^* d_v \\ \hline c_v a_w + d_v c_w - c_w a_v - d_w c_v & -c_v c_w^* + d_v d_w + c_w c_v^* - d_w d_v \end{array}$$

$$= \begin{array}{c|c} a_v \times a_w + c_w^* c_v - c_v^* c_w & 0 \\ \hline 0 & d_v \times d_w + c_w c_v^* - c_v c_w^* \end{array} = \text{RHS}$$

$$P'_w\,\varphi = \underbrace{I - P}\,\underline{\pi}_w\,\varphi = \underbrace{I - P}\,\underbrace{\alpha_w \varphi + \overset{*}{\beta}_w \varphi + \partial_w \varphi}$$

$$P'_w = Q_w + Q^*_w$$

$$Q_w\,\varphi = \underbrace{I - P}\,\underbrace{\alpha_w \varphi + \partial_w \varphi}$$

$$Q^*_w\,\varphi = \underbrace{I - P}\,\overset{*}{\beta}_w\,\varphi$$

$$P\,\overbrace{\underbrace{\alpha_w + \beta_w + \partial_w}\varphi} = 0$$

$$P\,\overbrace{\partial_w\varphi} = -P\,\overbrace{\underbrace{\alpha_w + \beta_w}\varphi}$$

$$0 = \underbrace{\underline{\pi}_w\varphi} \bowtie \psi + \varphi \bowtie \underbrace{\underline{\pi}_w\psi} = \underbrace{\alpha_w\varphi + \beta^*_w\varphi + \partial_w\varphi} \bowtie \psi + \varphi \bowtie \underbrace{\alpha_w\psi + \beta^*_w\psi + \partial_w\psi}$$

$$= \overbrace{\alpha_w\varphi + \partial_w\varphi} \bowtie \psi + \varphi \bowtie \overbrace{\beta_w\psi} + \overbrace{\beta_w\varphi} \bowtie \psi + \varphi \bowtie \overbrace{\alpha_w\psi + \partial_w\psi} = \overbrace{\alpha_w\varphi + \beta_w\varphi + \partial_w\varphi}^{\text{w-lin}} \bowtie \psi + \varphi \bowtie \overbrace{\alpha_w\psi + \beta_w\psi + \partial_w\psi}^{\text{w-lin}}$$

$$\Rightarrow \overbrace{\alpha_w\varphi + \beta_w\varphi + \partial_w\varphi} \bowtie \psi = 0 = \varphi \bowtie \overbrace{\alpha_w\psi + \beta_w\psi + \partial_w\psi}$$

$$\Rightarrow \alpha_w\varphi + \beta_w\varphi + \partial_w\varphi \in P^{\perp} \ni \alpha_w\psi + \beta_w\psi + \partial_w\psi$$

$${}^x\delta_g \; {}^{x \rtimes g}_{w \rtimes g}P_{y \rtimes g} \; {}^{y*}\delta_g = {}^x_wP_y$$

$${}^x\delta_w = {}^x\delta_{g_w}$$

$${}^x\delta_w \; {}^{x \rtimes g_w}_{w}P_{y \rtimes g_w} \; {}^{y*}\delta_w = {}^xP_y$$

$$-{}^x_wP'_y = {}^xP_y \underbrace{{}^x\delta'_w + {}^{y*}\delta'_w} + \underbrace{x \bowtie \gamma_w}\,{}^x\underline{P}_y + \overline{\underbrace{y \bowtie \gamma_w}\,{}^y\underline{P}_x}$$

$${}^x\delta'_w\,{}^xP_y + {}^x_wP'_y + \underbrace{x \bowtie \gamma_w}\,{}^x\underline{P}_y + \overline{\underbrace{y \bowtie \gamma_w}\,{}^y\underline{P}_x} + {}^xP_y\,{}^{y*}\delta'_w = 0$$

$${}^{x \rtimes g_w}\daleth = \int_{dy}^{\hbar} {}^{x \rtimes g_w}P_y \; {}^y\daleth$$

$$\overbrace{x \bowtie \gamma_w}\,{}^x\underline{\daleth} = \int_{dy}^{\hbar} \underbrace{\overbrace{x \bowtie \gamma_w}\,{}^x\underline{P}_y}\; {}^y\daleth$$

$$\overbrace{x \bowtie \gamma^{\ddagger}_w}\,{}^x\underline{\daleth} = \int_{dy}^{\hbar} \overline{\underbrace{y \bowtie \gamma_w}\,{}^y\underline{P}_x}\; {}^y\daleth$$