

$$P'_w = Q_w + \overset{*}{Q}_w$$

$$\underbrace{Q_z \varphi \bowtie Q_w \psi} - \underbrace{\overset{*}{Q}_w \varphi \bowtie \overset{*}{Q}_z \psi} = \underbrace{\varphi \bowtie \psi}_{z \bowtie w}$$

$$\underbrace{Q_z \varphi \bowtie \overset{*}{Q}_w \psi} = \underbrace{Q_w \varphi \bowtie \overset{*}{Q}_z \psi}$$

$$\begin{aligned} \underbrace{\varphi \bowtie \psi}_{z \bowtie w} - \underbrace{w \bowtie z} &= \varphi \bowtie \overbrace{P'_z P'_w - P'_w P'_z} \psi = \varphi \bowtie \underbrace{P'_z P'_w} \psi - \varphi \bowtie \underbrace{P'_w P'_z} \psi \\ &= \underbrace{P'_z \varphi \bowtie P'_w \psi} - \underbrace{P'_w \varphi \bowtie P'_z \psi} = \overbrace{Q_z + \overset{*}{Q}_z} \varphi \bowtie \overbrace{Q_w + \overset{*}{Q}_w} \psi - \overbrace{Q_w + \overset{*}{Q}_w} \varphi \bowtie \overbrace{Q_z + \overset{*}{Q}_z} \psi \\ &= \underbrace{Q_z \varphi \bowtie Q_w \psi} - \underbrace{Q_w \varphi \bowtie Q_z \psi} + \underbrace{Q_z \varphi \bowtie \overset{*}{Q}_w \psi} - \underbrace{Q_w \varphi \bowtie \overset{*}{Q}_z \psi} \\ &\quad + \underbrace{\overset{*}{Q}_z \varphi \bowtie Q_w \psi} - \underbrace{\overset{*}{Q}_w \varphi \bowtie Q_z \psi} + \underbrace{\overset{*}{Q}_z \varphi \bowtie \overset{*}{Q}_w \psi} - \underbrace{\overset{*}{Q}_w \varphi \bowtie \overset{*}{Q}_z \psi} \\ &\Rightarrow \underbrace{\varphi \bowtie \psi}_{z \bowtie w} = \underbrace{Q_z \varphi \bowtie Q_w \psi} - \underbrace{\overset{*}{Q}_w \varphi \bowtie \overset{*}{Q}_z \psi} \\ &\quad - \underbrace{\varphi \bowtie \psi}_{w \bowtie z} = \underbrace{\overset{*}{Q}_z \varphi \bowtie \overset{*}{Q}_w \psi} - \underbrace{Q_w \varphi \bowtie Q_z \psi} \\ \underbrace{Q_z \varphi \bowtie \overset{*}{Q}_w \psi} - \underbrace{Q_w \varphi \bowtie \overset{*}{Q}_z \psi} &= 0 = \underbrace{\overset{*}{Q}_z \varphi \bowtie Q_w \psi} - \underbrace{\overset{*}{Q}_w \varphi \bowtie Q_z \psi} \end{aligned}$$

$$\begin{aligned} \underbrace{\varphi \bowtie \psi}_{z \bowtie w} &= \underbrace{P'_z \varphi \bowtie P'_w \psi} = \overbrace{Q_z + \overset{*}{Q}_z} \varphi \bowtie \overbrace{Q_w + \overset{*}{Q}_w} \psi \\ &= \underbrace{Q_z \varphi \bowtie Q_w \psi} + \underbrace{Q_z \varphi \bowtie \overset{*}{Q}_w \psi} + \underbrace{\overset{*}{Q}_z \varphi \bowtie Q_w \psi} + \underbrace{\overset{*}{Q}_z \varphi \bowtie \overset{*}{Q}_w \psi} \\ &\Rightarrow \underbrace{\varphi \bowtie \psi}_{z \bowtie w} = \underbrace{Q_z \varphi \bowtie Q_w \psi} \end{aligned}$$

$$\underbrace{Q_z \varphi \bowtie \overset{*}{Q}_w \psi} = \underbrace{\overset{*}{Q}_z \varphi \bowtie Q_w \psi} = \underbrace{\overset{*}{Q}_z \varphi \bowtie \overset{*}{Q}_w \psi} = 0$$