$$0 \leqslant A \leqslant I$$

$$\text{commutant } A' = \frac{X \in {}^d\mathbb{K}_d}{XA = AX} \sqsubset {}^d\mathbb{K}_d$$

$$\text{bi-commutant } A'' = \frac{X \in {}^d\mathbb{K}_d}{X \times A' = 0} \sqsubset {}^d\mathbb{K}_d$$

$$A'' \sqsubset A'$$

$$X \in A'' \underset{Z = A}{\Longrightarrow} AX = XA$$

$$A'' \text{ commutativ}$$

$$X{:}Y \in A''$$

$$YA = AY \underset{Z = Y}{\Longrightarrow} YX = XY$$

$$\text{voll } \hbar = \frac{X \in A''}{0 \leqslant X \leqslant A} \sqsubset A''$$

$$X \in \hbar \xrightarrow{\wr} \hbar \ni {}^X\wr = \frac{A + X^2}{2}$$

$$ZA = AZ \Rightarrow ZX = XZ \Rightarrow ZX^2 = ZXX = XZX = XXZ = X^2Z$$

$$\Rightarrow Z\,{}^X\wr = \frac{ZA}{2} + \frac{ZX^2}{2} = \frac{AZ}{2} + \frac{X^2Z}{2} = {}^X\wr\, Z \Rightarrow {}^X\wr \in A''$$

$$0 \leqslant {}^X\wr = \frac{A + X^2}{2} \leqslant \frac{A + X}{2} \leqslant \frac{A + A}{2} = A$$

$$\hbar \xrightarrow[\operatorname{contr}_{\|A\|}]{\wr} \hbar$$

$$\underset{\text{Ban}}{\Longrightarrow} \bigvee_{B}^{\hbar} B = {}^{B}\wr$$

$$X \in \hbar \ni Y$$

$${}^{X}\wr - {}^{Y}\wr = \frac{A + X^2}{2} - \frac{A + Y^2}{2} = \frac{X^2 - Y^2}{2} \underset{XY=YX}{\equiv} \frac{(X+Y)(X-Y)}{2}$$

$$\Longrightarrow \left\| {}^{X}\wr - {}^{Y}\wr \right\| \leqslant \frac{1}{2} \|X+Y\| \|X-Y\|$$

$$\begin{cases} X \leqslant A \\ Y \leqslant A \end{cases} \Longrightarrow \frac{X+Y}{2} \leqslant A \Longrightarrow \frac{1}{2}\|X+Y\| \leqslant \|A\| < 1$$

$$\overparen{I - B}^{2} = I - A$$

$$B = {}^{B}\wr = \frac{A + B^2}{2} \Longrightarrow 2B = A + B^2 \Longrightarrow \overparen{I - B}^{2} = I - 2B + B^2 = I - A$$