

$${}_{\zeta}Q \blacktriangleright a = \zeta | a \bar{\zeta} : \quad {}_{\zeta}J = {}^{\zeta}Q \eta$$

$${}_{w\bar{\varepsilon}_I} \iota_{\varepsilon_I} = w \blacktriangleright x \text{ mult}$$

$$\underbrace{w\bar{\varepsilon}_I | \zeta}_{\zeta} \underbrace{\varepsilon_I | \zeta}_{\zeta} = \underbrace{w\bar{\zeta} | \varepsilon_I}_{\zeta} \underbrace{\varepsilon_I | \zeta}_{\zeta} = \underbrace{w\bar{\zeta} | \zeta}_{\zeta} = w \blacktriangleright {}^{\zeta}Q = w \blacktriangleright x$$

$$\underbrace{\iota_{w\bar{\varepsilon}_I} \iota_{\varepsilon_I}}_{\zeta} J = \underbrace{w\bar{\varepsilon}_I | \zeta}_{\zeta} \underbrace{\varepsilon_I | \zeta}_{\zeta} {}_{\zeta}J = w \blacktriangleright x {}^{\zeta}Q \eta$$

$$\iota_{w\varepsilon_I} \partial_{\bar{\varepsilon}_I} + \partial_{w\varepsilon_I} \iota_{\bar{\varepsilon}_I} = 4 \partial_{x\dot{u}w}$$

$$\underbrace{\partial_{\beta} J}_{\zeta} = \beta {}_{\zeta}J = \beta {}_{\zeta}Q {}^{\zeta}Q \eta$$

$${}_{\zeta}Q = \overline{{}_{\zeta}Q \blacktriangleright e_i} e_i = \zeta | e_i \bar{\zeta} e_i \Rightarrow \alpha {}_{\zeta}Q = \overline{\alpha | e_i \bar{\zeta} + \zeta | e_i \bar{\alpha}} e_i = 2 \alpha | e_i \bar{\zeta} e_i$$

$$\underbrace{\iota_{\alpha} \partial_{\beta} + \partial_{\alpha} \iota_{\beta}}_{\zeta} J = \overline{\alpha | \zeta} \underbrace{\beta {}_{\zeta}J}_{\zeta} + \overline{\beta | \zeta} \underbrace{\alpha {}_{\zeta}J}_{\zeta} = \overline{\alpha | \zeta} \underbrace{\beta {}_{\zeta}Q}_{\zeta} + \overline{\beta | \zeta} \underbrace{\alpha {}_{\zeta}Q}_{\zeta} {}^{\zeta}Q \eta$$

$$= \overline{\alpha | \zeta} \underbrace{\beta {}_{\zeta}Q \blacktriangleright e_i}_{\zeta} + \overline{\beta | \zeta} \underbrace{\alpha {}_{\zeta}Q \blacktriangleright e_i}_{\zeta} e_i {}^{\zeta}Q \eta = 2 \overline{\alpha | \zeta} \underbrace{\beta | e_i \bar{\zeta}}_{\zeta} + \overline{\beta | \zeta} \underbrace{\alpha | e_i \bar{\zeta}}_{\zeta} e_i {}^{\zeta}Q \eta$$

$$\underbrace{x\dot{u}w}_{\zeta} \blacktriangleright b = w \bar{\zeta} | b \bar{\zeta}$$

$$2 \text{ LHS} = \underbrace{x\dot{u}w}_{\zeta} + \underbrace{w\dot{u}x}_{\zeta} \blacktriangleright b = \zeta \zeta^* w + w \zeta \zeta^* \blacktriangleright b = 2w \bar{\zeta} | b \bar{\zeta}$$

$$\underbrace{w\varepsilon_I | \zeta}_{\zeta} \underbrace{\bar{\varepsilon}_I | e_i \bar{\zeta}}_{\zeta} + \underbrace{\bar{\varepsilon}_I | \zeta}_{\zeta} \underbrace{w\varepsilon_I | e_i \bar{\zeta}}_{\zeta} = \underbrace{w\bar{\zeta} | \bar{\varepsilon}_I}_{\zeta} \underbrace{\bar{\varepsilon}_I | e_i \bar{\zeta}}_{\zeta} + \underbrace{\bar{\varepsilon}_I | \zeta}_{\zeta} \underbrace{w\bar{\varepsilon}_I | e_i \bar{\zeta}}_{\zeta} = \underbrace{w\bar{\zeta} | e_i \bar{\zeta}}_{\zeta} + \underbrace{w\bar{\varepsilon}_I | \zeta}_{\zeta} = 2 \underbrace{w\bar{\zeta} | e_i \bar{\zeta}}_{\zeta} = 2 \underbrace{x\dot{u}w}_{\zeta} \blacktriangleright e_i$$

$$\underbrace{\iota_{w\varepsilon_I} \partial_{\bar{\varepsilon}_I} + \partial_{w\varepsilon_I} \iota_{\bar{\varepsilon}_I}}_{\zeta} J = 2 \underbrace{w\varepsilon_I | \zeta}_{\zeta} \underbrace{\bar{\varepsilon}_I | e_i \bar{\zeta}}_{\zeta} + \underbrace{\bar{\varepsilon}_I | \zeta}_{\zeta} \underbrace{w\varepsilon_I | e_i \bar{\zeta}}_{\zeta} e_i {}^{\zeta}Q \eta = 4 \underbrace{x\dot{u}w}_{\zeta} \blacktriangleright e_i e_i {}^{\zeta}Q \eta = 4 \underbrace{x\dot{u}w}_{\zeta} {}^{\zeta}Q \eta = 4 \partial_{x\dot{u}w} {}^{\zeta}Q \eta$$

$$\underbrace{w\varepsilon_I \mathfrak{X} \bar{\varepsilon}_I}_{\zeta} Q^{\zeta} \gamma = 2a \partial_w \gamma$$

$$\overline{\partial_w \gamma}^x = w^x \gamma = w \mathfrak{X} e_i e_i^x \gamma$$

$$\underbrace{\alpha \mathfrak{X} \beta}_{\zeta} Q = 2 \underbrace{\alpha | e_i \bar{\beta}}_{\zeta} e_i$$

$$w \varepsilon_I | b \varepsilon_I = \overline{b w} \varepsilon_I | \varepsilon_I = \text{tr}_E \overline{b} w = a w \mathfrak{X} b$$

$$\underbrace{w \varepsilon_I \mathfrak{X} \bar{\varepsilon}_I}_{\zeta} Q = 2 \underbrace{w \varepsilon_I | e_i \varepsilon_I}_{\zeta} e_i = 2a \underbrace{w \mathfrak{X} e_i}_{\zeta} e_i = 2aw \Rightarrow \text{LHS} = 2aw \zeta \gamma = \text{RHS}$$

$$\overline{w\varepsilon_{I\zeta}Q} \mathfrak{X} \overline{\varepsilon_{I\zeta}Q} \zeta^Q \eta = 4w \mathfrak{X} \zeta^Q \overline{P_\partial \eta}$$

$$w \mathfrak{X} \overline{e_i^* x e_j} = \overline{w e_i^* x} \mathfrak{X} e_j = \overline{x e_i^* w} \mathfrak{X} e_j = x \mathfrak{X} \overline{e_i^* w e_j}$$

$$w \mathfrak{X}^x \overline{P_\partial \eta} = \overline{w \mathfrak{X} e_i^* x e_j} \overline{e_i \mathfrak{X} e_j^x \eta} = \overline{x \mathfrak{X} e_i^* w e_j} \overline{e_i \mathfrak{X} e_j^x \eta}$$

$$\overline{awa} \zeta = \overline{a w a} \zeta$$

$$2awa = \overline{awa} + \overline{aw} + \overline{wa} + \overline{aw}a - \overline{aw} + \overline{wa}$$

$$\begin{aligned} 2 \text{ LHS} &= \overline{awa} + \overline{aw} + \overline{wa} + \overline{aw}a \zeta - \overline{aw} + \overline{wa} \zeta = \overline{awa} + \overline{aw} + \overline{wa} + \overline{aw}a \zeta - \overline{aw} + \overline{wa} \zeta \\ &= \overline{awa} + \overline{aw} + \overline{wa} + \overline{aw}a \zeta - \overline{aw} + \overline{wa} \zeta = 2 \text{ RHS} \end{aligned}$$

$$\zeta | \overline{a^* w a} \bar{\zeta} = \zeta | \overline{a^* w a} \bar{\zeta} = \bar{a} \zeta | \overline{w a} \bar{\zeta} = w \bar{a} \zeta | a \bar{\zeta}$$

$$\zeta | \overline{a^* w b} \bar{\zeta} = w \bar{a} \zeta | b \bar{\zeta}$$

$$\zeta | \overline{e_i^* w e_j} \bar{\zeta} = w \bar{e}_i \zeta | e_j \bar{\zeta}$$

$$\overline{w\varepsilon_I | e_i \bar{\zeta}} \overline{\varepsilon_I | e_j \bar{\zeta}} = \overline{w \bar{e}_i \zeta | \varepsilon_I} \overline{\varepsilon_I | e_j \bar{\zeta}} = w \bar{e}_i \zeta | e_j \bar{\zeta}$$

$$\overline{w\varepsilon_{I\zeta}Q} \mathfrak{X} \overline{\varepsilon_{I\zeta}Q} = 4 \overline{w\varepsilon_I | e_i \bar{\zeta}} \overline{\varepsilon_I | e_j \bar{\zeta}} e_i \mathfrak{X} e_j = 4 \overline{w \bar{e}_i \zeta | e_j \bar{\zeta}} e_i \mathfrak{X} e_j = 4 \zeta | \overline{e_i^* w e_j} \bar{\zeta} e_i \mathfrak{X} e_j = 4 \overline{Q \mathfrak{X} e_i^* w e_j} e_i \mathfrak{X} e_j$$

$$\text{LHS} = 4 \overline{Q \mathfrak{X} e_i^* w e_j} e_i \mathfrak{X} e_j \zeta^Q \eta = 4 \overline{Q \mathfrak{X} e_i^* w e_j} e_i \mathfrak{X} e_j \zeta^Q \eta = \text{RHS}$$

$$\partial_{w\varepsilon_I} \partial_{\bar{\varepsilon}_I} = 2a \partial_w + 4w \mathfrak{K} P_\partial$$

$$\overbrace{\partial_\alpha \partial_\beta \mathbf{J}}^\zeta = \overbrace{\alpha \mathfrak{X} \beta}_\zeta \mathbf{J} = \overbrace{\alpha \mathfrak{X} \beta}_\zeta \overbrace{Q}_{\zeta^-} \zeta^Q \mathbf{J} + \overbrace{\alpha \overbrace{Q}_{\zeta^-} \mathfrak{X} \beta}_{\zeta^-} \overbrace{Q}_{\zeta^-} \zeta^Q \mathbf{J}$$

$$\overbrace{\partial_{w\varepsilon_I} \partial_{\bar{\varepsilon}_I} \mathbf{J}}^\zeta = \overbrace{w\varepsilon_I \mathfrak{X} \bar{\varepsilon}_I}_\zeta \overbrace{Q}_{\zeta^-} \zeta^Q \mathbf{J} + \overbrace{w\varepsilon_I \overbrace{Q}_{\zeta^-} \mathfrak{X} \bar{\varepsilon}_I}_{\zeta^-} \overbrace{Q}_{\zeta^-} \zeta^Q \mathbf{J} = 2a \partial_w \mathbf{J} + 4w \mathfrak{K} \overbrace{\zeta^Q P_\partial}^\zeta \mathbf{J}$$

$$\overbrace{\iota_{w\bar{\varepsilon}_I} - \partial_{w\bar{\varepsilon}_I}} \overbrace{\iota_{\varepsilon_I} - \partial_{\varepsilon_I}} = w \mathfrak{K} x + 2a \partial_w - 4 \partial_{x \ddot{u} w} + 4w \mathfrak{K} P_\partial$$

$$\begin{aligned} \overbrace{\iota_{w\bar{\varepsilon}_I} - \partial_{w\bar{\varepsilon}_I}} \overbrace{\iota_{\varepsilon_I} - \partial_{\varepsilon_I}} \mathbf{J} &= \overbrace{\iota_{w\bar{\varepsilon}_I}} \overbrace{\iota_{\varepsilon_I}} \mathbf{J} + \overbrace{\partial_{w\bar{\varepsilon}_I}} \overbrace{\partial_{\varepsilon_I}} \mathbf{J} - \overbrace{\iota_{w\bar{\varepsilon}_I} \partial_{\varepsilon_I} + \partial_{w\bar{\varepsilon}_I} \iota_{\varepsilon_I}} \mathbf{J} \\ &= w \mathfrak{K} x^x \mathbf{J} + 2a \partial_w^x \mathbf{J} - 4 \partial_{x \ddot{u} w}^x \mathbf{J} + 4w \mathfrak{K} \overbrace{P_\partial}^x \mathbf{J} \end{aligned}$$